Section 4 – Topic 7
Solving Quadratic Equations by Completing the Square

Recall perfect square trinomials.

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

We can use this to solve quadratic equations by completing the square.

Solve $2x^2 - 5x + 2 = 0$.

$-2 \quad -2$

- Write the quadratic equation in the form $ax^2 + bx = c$.

$$2x^2 - 5x = -2$$

- If a does not equal 1, divide every term by a.

$$x^2 - \frac{5}{2}x = -1$$

- Divide b by 2 and square the result.

$$\left(-\frac{5}{2} \div 2\right)^2 = \left(-\frac{5}{2} \cdot \frac{1}{2}\right)^2 = \left(-\frac{5}{4}\right)^2 = \frac{25}{16}$$

- Add that number to both sides of the equation.

$$\frac{-1}{1} = \frac{-16}{16}$$

$$x^2 - \frac{5}{2}x + \frac{25}{16} = -1 + \frac{25}{16} - \frac{16}{16}$$

$$x^2 - \frac{5}{2}x + \frac{25}{16} = \frac{9}{16}$$

- Factor the trinomial.

$$\left(x - \frac{5}{4}\right)^2 = \frac{9}{16}$$

- Take the square root of both sides.

$$\sqrt{\left(x - \frac{5}{4}\right)^2} = \sqrt{\frac{9}{16}} \rightarrow x - \frac{5}{4} = \pm\frac{3}{4}$$

- Solve for x.

$$x - \frac{5}{4} = \frac{3}{4} + \frac{5}{4} \qquad x - \frac{5}{4} = -\frac{3}{4} + \frac{5}{4}$$

$$x = \frac{8}{4} = 2 \qquad x = \frac{2}{4} = \frac{1}{2}$$

$$x = 2, \frac{1}{2}$$

Let's Practice!

$\left(-\frac{5}{2} \div 2\right)^2$

1. Complete the square to solve the following equation.

$$2x^2 - 5x + 5 = 0$$

$\left(-\frac{5}{2} \cdot \frac{1}{2}\right)^2 = \left(-\frac{5}{4}\right)^2 = \frac{25}{16}$

$2x^2 - 5x = -5$

$-\frac{5}{2} = \frac{-40}{16}$

$x^2 - \frac{5}{2}x = -\frac{5}{2}$

$x^2 - \frac{5}{2}x + \frac{25}{16} = -\frac{5}{2} + \frac{25}{16} - \frac{40}{16}$

$x^2 - \frac{5}{2}x + \frac{25}{16} = \frac{-15}{16}$

$x - \frac{5}{4} = \pm\frac{i\sqrt{15}}{4}$

$\sqrt{\left(x - \frac{5}{4}\right)^2} = \sqrt{\frac{-15}{16}}$ $\quad x - \frac{5}{4} = \pm\frac{\sqrt{-15}}{4} \Rightarrow$ $\quad x = \frac{5}{4} \pm \frac{i\sqrt{15}}{4}$

Try it!

2. Complete the square to solve the following equation.

$$-3x^2 - 7x + 3 = 0$$

$\left(\frac{7}{3} \div 2\right)^2$

$\left(\frac{7}{3} \cdot \frac{1}{2}\right)^2 = \left(\frac{7}{6}\right)^2 = \frac{49}{36}$

$-3x^2 - 7x = -3$ $\quad (b \div 2)^2$

$x^2 + \frac{7}{3}x = 1$

$x^2 + \frac{7}{3}x + \frac{49}{36} = \frac{49}{36} + \frac{36}{36}$

$x^2 + \frac{7}{3}x + \frac{49}{36} = \frac{85}{36}$

$\sqrt{\left(x + \frac{7}{6}\right)^2} = \pm\sqrt{\frac{85}{36}} \rightarrow x + \frac{7}{6} = \pm\frac{\sqrt{85}}{6}$

$x = -\frac{7}{6} \pm \frac{\sqrt{85}}{6}$

BEAT THE TEST!

1. Graduate students throw their mortarboards (graduation caps) into the air to convey their hopes and aspirations for a future career. The height $h(t)$, in feet, of a mortarboard in the air t seconds after it is thrown can be modeled by the function $h(t) = -16t^2 + 32t + 4$.

   Part A: Edina was investigating the time it takes a mortarboard to hit the ground after being thrown. She completed the square to find the solution. Before getting to the solution, she arrived at the following equation:

   $$(t - h)^2 = 1.25$$

   What is the value of h?

   $h =$ [1] .

   Part B: How much time does it take a mortarboard to hit the ground after being thrown?

   [2.1] seconds.

$\left(\frac{-2}{2}\right)^2$

$(-1)^2 = 1$

$-16t^2+32t+4=0$

$\frac{-16t^2}{-16} + \frac{32t}{-16} = \frac{-4}{-16}$

$t^2 - 2t = -\frac{1}{4}$

$t^2-2t+1 = -.25+1$

$\sqrt{(t-1)^2} = \sqrt{1.25}$

$t-1 = \pm 1.1$

$+1 \quad +1$

$t = 1.1+1 \qquad t = -1.1+1$

$t = 2.1 \qquad = -.1$

2. A pair of data analysts for a college track and field team measured the height of one of their high jumpers in the air t seconds after he takes off. The analysts modeled it with the function $h(t) = -16t^2 + 48t$ and attempted to calculate the amount of time it takes him to land on the 4 feet tall landing pit. The calculations are shown below.

Data Analyst #1		**Data Analyst #2**
$-16t^2 + 48t = -4$	Step 1	$-16t^2 + 48t = 4$
$t^2 - 3t = 0.25$	Step 2	$t^2 + 3t = -0.25$
$t^2 - 3t + 2.25 = 2.5$	Step 3	$t^2 + 3t + 2.25 = 2$
$(t - 1.5)^2 = 2.5$	Step 4	$(t + 1.5)^2 = 2$
$t = 3.08$	Step 5	$t = 0$
4.06 seconds		0 seconds

$\left(-\frac{3}{2}\right)^2 = (-1.5)^2 = 2.25$

$t^2 - 3t = -0.25$

$t^2 - 3t + 2.25 = 2$

$(t - 1.5)^2 = 2$

$t - 1.5 = \pm\sqrt{2}$

Part A: Which of the following statements is correct?

- Ⓐ Both data analysts did Step 1 wrong. ✓
- Ⓑ Both data analysts did Step 4 wrong.
- ● Data Analyst #1 did Step 1 wrong, whereas Data Analyst #2 did Step 2 wrong. ✓
- Ⓓ Both data analysts did Step 2 wrong.

$t = \sqrt{2} + 1.5$

$= 1.4 + 1.5 = 2.9$

Part B: What is the amount of time it takes the high jumper to land on the landing pit?

$t = -\sqrt{2} + 1.5$

$-1.4 + 1.5 = .1$

[2.9] seconds.