Section 4 – Topic 8
Solving Quadratic Equations Using the Quadratic Formula – Part 1

$\left(\frac{b}{a} \cdot \frac{1}{2}\right)^2$

$\left(\frac{b}{2a}\right)^2$

Consider a quadratic equation in standard form.

$\sqrt{4a^2} = 2a$

$$y = ax^2 + bx + c$$

$ax^2 + bx = -c$

We can use completing the square to derive the quadratic formula.

$x^2 + \frac{b}{a}x = -\frac{c}{a}$

$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \left(-\frac{c}{a}\right) + \frac{b^2}{4a^2}$ (4a)

$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2}$

$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$

$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

When must you use completing the square or the quadratic formula to solve a quadratic?

when the quadratic can't be factored

Let's Practice!

1. Solve the following equations using the quadratic formula.

   a. $\overset{a}{\frac{1}{4}x^2} \overset{b}{- x} + \overset{c}{\frac{1}{2}} = 0$

$$\frac{1 \pm \sqrt{(-1)^2 - 4(\frac{1}{4})(\frac{1}{2})}}{2(\frac{1}{4})}$$

$$\frac{1 \pm \sqrt{1 - \frac{1}{2}}}{\frac{1}{2}}$$

$$\frac{\left(1 \pm \sqrt{\frac{1}{2}}\right)2}{\frac{1}{2} \cdot 2}$$

$$\boxed{2 \pm 2\sqrt{\frac{1}{2}}}$$

b. $(a+2)^2 = 2a$

$(a+2)(a+2) = 2a$

$a^2 + 2a + 2a + 4 = 2a$

$a^2 + 4a + 4 = 2a$

$a^2 + 2a + 4 = 0$

$$\frac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$$

$$\frac{-2 \pm \sqrt{4 - 16}}{2}$$

$$\frac{2 \pm \sqrt{-12}}{2}$$

$$\frac{2 \pm 2i\sqrt{3}}{2}$$

$1 \pm i\sqrt{3}$

$\sqrt{-12}$

$i\sqrt{12}$

$i\sqrt{4}\sqrt{3}$

$2i\sqrt{3}$

Try it!

2. Solve the following equations using the quadratic formula.

a. $\frac{2}{5}m^2 + \frac{1}{5}m + 3 = 0$

$$\frac{-\frac{1}{5} \pm \sqrt{\left(\frac{1}{5}\right)^2 - 4\left(\frac{2}{5}\right)(3)}}{2\left(\frac{2}{5}\right)}$$

$$\frac{-\frac{1}{5} \pm \sqrt{\frac{1}{25} - \frac{24}{5}\cdot\frac{120}{25}}}{\frac{4}{5}}$$

$$\frac{-\frac{1}{5} \pm \sqrt{\frac{-119}{25}}}{4/5}$$

$$\left(\frac{-\frac{1}{5} \pm \frac{i\sqrt{119}}{5}}{\frac{4}{5}}\right)\frac{5}{4}$$

$$\frac{-1 \pm i\sqrt{119}}{4}$$

b. $\left(b-\frac{1}{2}\right)^2=\frac{b}{2}$

$\left(b-\frac{1}{2}\right)\left(b-\frac{1}{2}\right)=\frac{b}{2}$

$b^2-\frac{1}{2}b-\frac{1}{2}b+\frac{1}{4}=\frac{b}{2}$

$b^2-b+\frac{1}{4}=\frac{b}{2}$

$-\frac{b}{2} \qquad -\frac{b}{2}$

$b^2-\frac{3}{2}b+\frac{1}{4}=0$

$$\frac{\frac{3}{2}\pm\sqrt{\left(-\frac{3}{2}\right)^2-4(1)\left(\frac{1}{4}\right)}}{2(1)}$$

$$\frac{\frac{3}{2}\pm\sqrt{\frac{9}{4}-\cancel{4}\left(\frac{4}{4}\right)}}{2}=\frac{\frac{3}{2}\pm\sqrt{\frac{5}{4}}}{2}$$

$$\frac{\left(\frac{3}{2}\pm\frac{\sqrt{5}}{2}\right)\cdot\frac{1}{2}}{(2)\cdot\frac{1}{2}}=\boxed{\frac{3}{4}\pm\frac{\sqrt{5}}{4}}$$