Section 5 – Topic 13
Classifying Quadratic Functions and Finding Inverses

How can we determine that a function is even?

$f(x)=f(-x)$ symmetric over y-axis

How can we determine that a function is odd?

$f(x)=-f(x)$ symmetric about the origin

Let's Practice!

1. Quadratic functions are
   - ○ always
   - ● sometimes
   - ○ never

   even.

2. Quadratic functions are
   - ○ always
   - ○ sometimes
   - ● never

   odd.

Try It!

solutions = crosses the x-axis

3. Sketch the graphs of three even quadratic functions; one with two solutions, one with one solution, and one with no solutions.

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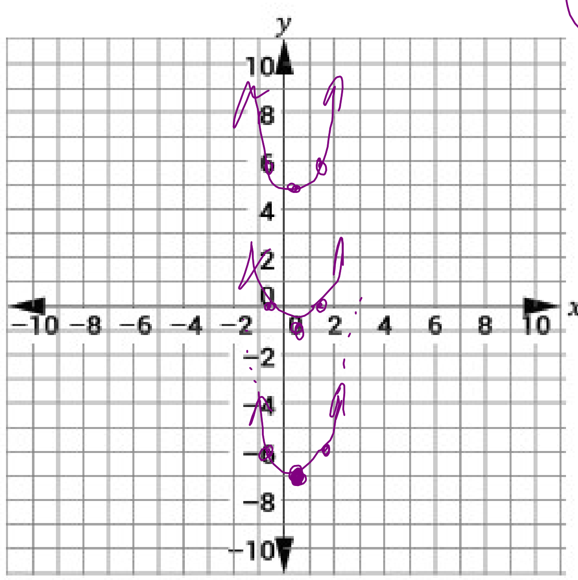


$y = x^2 - 6$
two solutions

$y = x^2$
one solution

$y = x^2 + 6$
no solution

4. Give algebraic representations of three even quadratic functions; one with two solutions, one with one solution, and one with no solutions.

How to determine the inverse of a function:

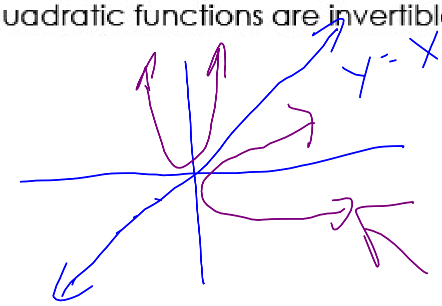
Step 1: Write function notation $f(x)$ as y.
Step 2: switch the variables y and x.
Step 3: solve the equation for y.
Step 4: Write in function notation $f^{-1}(x)$.

There are two ways to determine if two functions are inverses:

Algebraically: Functions $f(x)$ and $g(x)$ are inverses if
$f(g(x)) = g(f(x)) = x$

Graphically: Functions $f(x)$ and $g(x)$ are inverses if they are reflections over the line $y = x$.

Determine if quadratic functions are invertible. Justify your answer.



Does not pass the vertical line test

not a function

Let's Practice!

5. Consider the quadratic function $f(x) = (x+3)^2 + 2$. $(-3, 2)$

   a. Restrict the domain so that $f(x)$ is invertible.

   $f(x) = (x+3)^2 + 2 \quad x \leq -3$

   $f(x) = (x+3)^2 + 2 \quad x \geq -3$

   b. Find the inverse for each domain.

   $y = (x+3)^2 + 2$

   $x = (y+3)^2 + 2$

   $-2 \qquad -2$

   $\cancel{x}-2 = (y+3)^2$

   $\pm\sqrt{x-2} = y+3 \rightarrow -3 \pm \sqrt{x-2} = y$

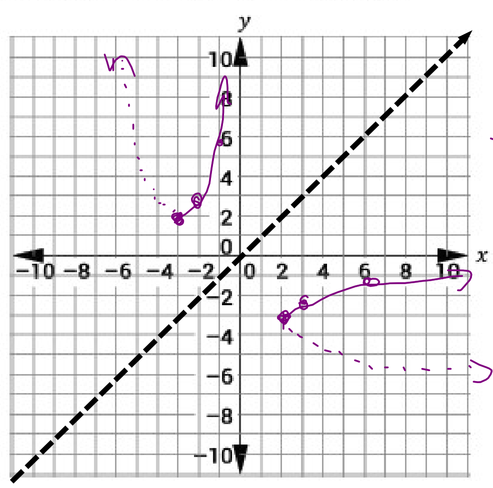
   $f^{-1}(x) = -3 \pm \sqrt{x-2}$

   c. Sketch the graph of the quadratic function with the restricted domains and its inverse.

$(-3, 2)$

$(-2, 3)$

$(-1, 6)$



$f^{-1}(x) = -3 - \sqrt{x-2} \quad x \leq -3$

$f^{-1}(x) = -3 + \sqrt{x-2} \quad x \geq -3$

Try It!

6. Consider the functions $g(x) = x^2 - 8x + 17$ for $x \geq 4$ and $h(x) = \sqrt{x-1} + 4$.

   a. Prove that $h(x)$ and $g(x)$ are inverses algebraically.

$$h(g(x)) = \sqrt{(x^2 - 8x + 17) - 1} + 4$$
$$= \sqrt{x^2 - 8x + 16} + 4$$
$$= \sqrt{(x-4)^2} + 4 \rightarrow x - 4 + 4 = x$$

   b. Show that $h(x)$ and $g(x)$ are inverses by graphing.

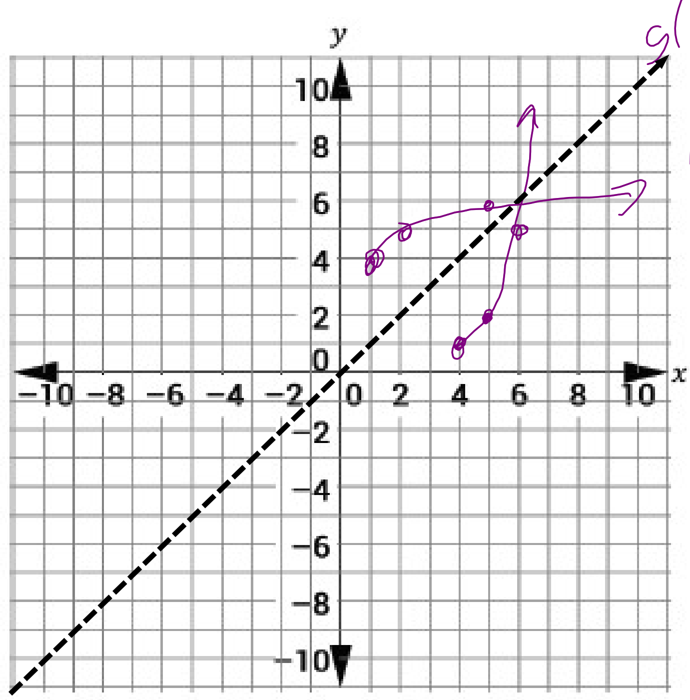
$g(x) = x^2 - 8x + 17 \quad x \geq 4$

$h(x) = \sqrt{x-1} + 4$

$g(x)$

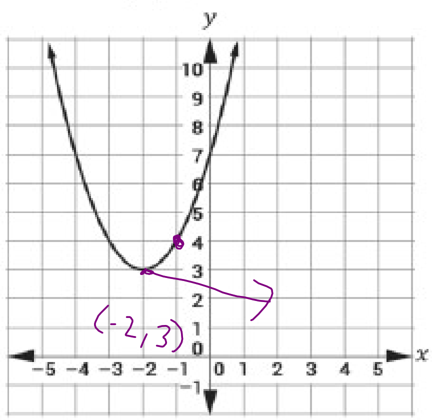
x	$g(x)$
4	1
5	2
6	5

x	$h(x)$
1	4
2	5
5	6



BEAT THE TEST!

1. A quadratic function $f(x)$ is shown.



Select symbols and values to restrict the domain of $f(x)$ so that $f^{-1}(x)$, is a function and the domain of $f(x)$ includes $x = -1$.