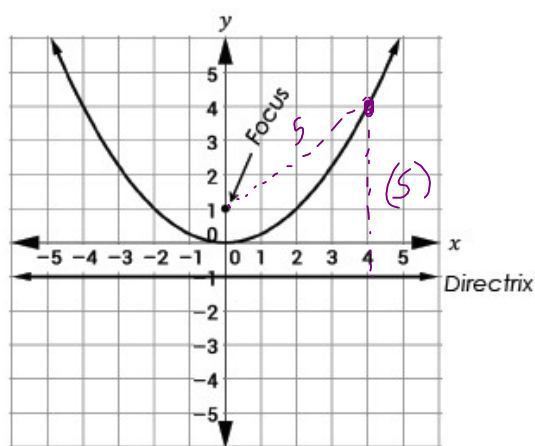## Section 5 – Topic 7
Writing Quadratic Equations when Given a Focus and Directrix

The following parabola represents a quadratic function.

Notice how the **focus** and **directrix** are labeled in the graph.

➤ A **parabola** is a set of points called a _locus_ of points.

➤ The _focus_ of a parabola is a fixed point on the interior of a parabola used in the formal definition of the curve.

➤ The _directrix_ of a parabola is a line perpendicular to the axis of symmetry used in the definition of a parabola.

Consider the point $(2, 1)$ on the parabola.

What is the distance from this point to the focus?

2 units

What is the distance from this point to the directrix?

2 units

Now consider the point $(4, 4)$ on the same parabola.

What is the distance from this point to the focus? *Hint: Use the distance formula.* $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

focus $(0, 1)$

$\sqrt{(4-0)^2 + (4-1)^2}$

$\sqrt{(4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

What is the distance from this point to the directrix?

$$(4,4) \qquad (4,-1)$$
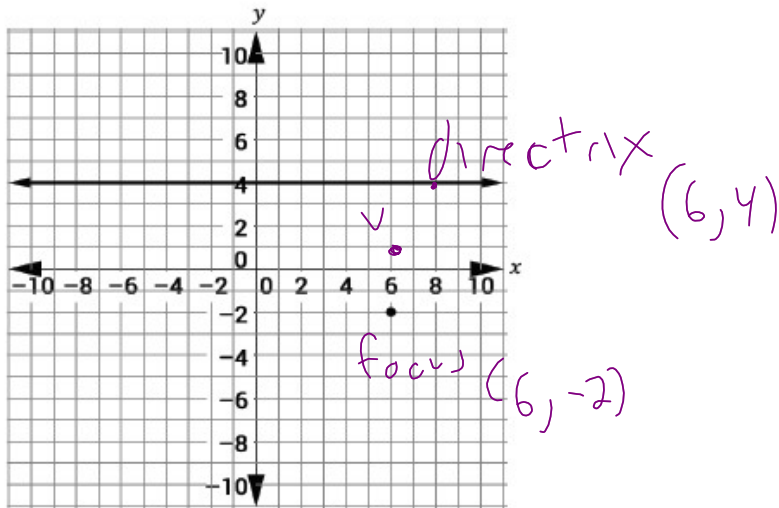
$$\sqrt{(4-4)^2 + (4+1)^2}$$

$$\sqrt{(5)^2}$$

$$\sqrt{25} = 5$$

What do you notice about the distances?

They are equal!

The graph below shows a focus and directrix for a specific parabola.

directrix (6,4)

focus (6,-2)

What formula from geometry can help us find the vertex of the parabola?
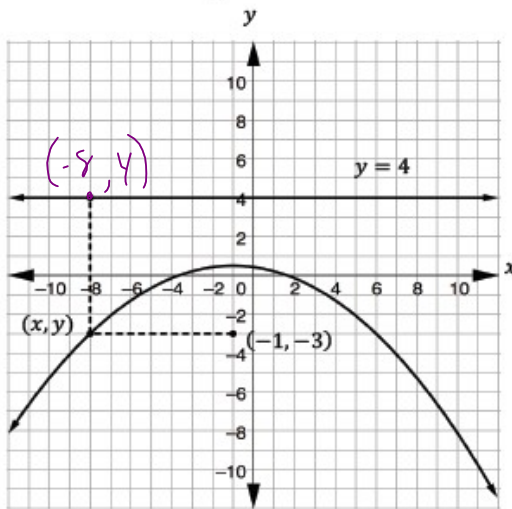
Midpoint formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

What is the vertex of the parabola? $\frac{6+6}{2} = 6$ $\frac{4+(-2)}{2} = \frac{2}{2} = 1$

(6, 1)

What other information would be needed to write the equation for the parabola?   any point (x,y) or a

Let's Practice!

1. Consider the following parabola with the focus at $(-1,-3)$ and the directrix line at $y = 4$.

a. Use the distance formula to describe the distance from (x, y) to the <u>focus</u>.

$$\sqrt{(x+1)^2 + (y+3)^2}$$

b. Use the distance formula to describe the distance from (x, y) to the directrix.

$$\sqrt{(x-x)^2 + (y-4)^2}$$

$$\sqrt{(y-4)^2}$$

c. What do you know about these two distances?

they are equal

d. Use this information to write the equation for the quadratic.

$$\sqrt{(x+1)^2+(y+3)^2} = \sqrt{(y-4)^2}$$

$(x+1)^2 + (y+3)^2 = (y-4)^2$

$x^2 + 2x + 1 + y^2 + 6y + 9 = y^2 - 8y + 16$

$x^2 + 2x + 10 + 6y = -8y + 16$
$\qquad -16 \qquad -6y \quad -6y \quad -16$

$\dfrac{x^2 + 2x - 6}{-14 \;-14 \;-14} = \dfrac{-14y}{-14}$

$-\dfrac{1}{14}x^2 - \dfrac{1}{7}x + \dfrac{3}{7} = y$

Try It!

2. A parabola has a focus on the origin and its directrix is $y = 7$.

What is the equation for the parabola?

$$\sqrt{(x-0)^2 + (y-0)^2} = \sqrt{(y-7)^2}$$

$x^2 + y^2 = y^2 - 14y + 49$

$x^2 = -14y + 49$
$\quad -49 \qquad\qquad -49$

$\dfrac{x^2 - 49}{14} = \dfrac{-14y}{-14}$

$-\dfrac{1}{14}x^2 + \dfrac{7}{2} = y$

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BEAT THE TEST!

1. In the diagram below, F is the focus of the parabola, line d is the directrix, and $\overline{AB} \perp d$.
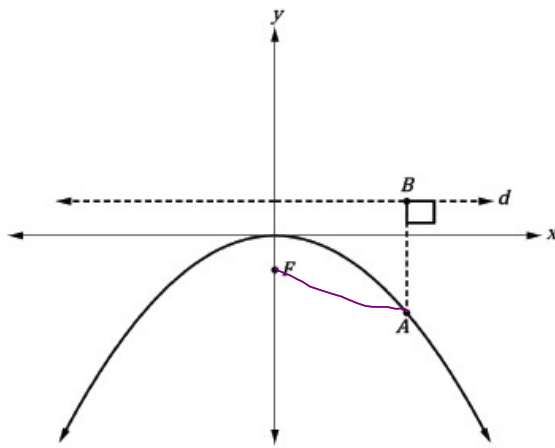
What is the relationship between FA and AB?

  Ⓐ $FA < AB$
  Ⓑ $FA = AB$
  © $FA > AB$
  Ⓓ   A relationship cannot be determined.

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2. A parabola has a focus of $(2,5)$ and the directrix is $y = 3$.

(x,y)

What is the equation for the parabola in standard form?

$$\sqrt{(x-2)^2 + (y-5)^2} = \sqrt{(y-3)^2}$$

$$x^2 - 4x + 4 + y^2 - 10y + 25 = y^2 - 6y + 9$$

$$x^2 - 4x + 29 - 10y = -6y + 9$$
$$-9 + 10y \qquad +10y \qquad -9$$

$$\frac{x^2}{4} - \frac{4x}{4} + \frac{20}{4} = \frac{4y}{4}$$

$$\boxed{\frac{1}{4}x^2 - x + 5 = y}$$