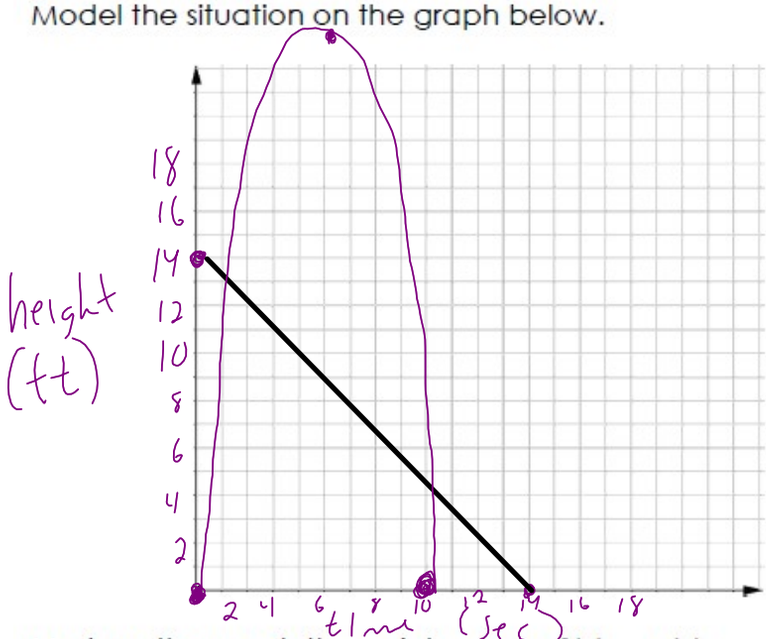
Section 5 – Topic 8
Systems of Equations with Quadratic Equations – Part 1

NASA launched a model rocket from the grounds of Cape Canaveral. The height, in feet, of the rocket with respect to time can be modeled by the function $h(x) = -x^2 + 10x$, where x is time, in seconds. At the same time, a Navy fleet shot a laser beam from a deck 14 feet above sea level. The laser beam follows a straight path represented by the equation $g(x) = -x + 14$.

$\frac{-b}{2a} = \frac{-10}{2(-1)} = 5$

$-(5)^2 + 10(5) = 25$

Model the situation on the graph below.



Mark on the graph the points where $f(x) = g(x)$.

What do these points represent? The points where the laser beam intersects the model rocket

The rocket and the laser beam model a quadratic-linear system of equations. We can also solve this system algebraically.

The following functions model the system.

$$f(x) = -x^2 + 10x$$
$$g(x) = -x + 14$$

One equation is a quadratic and has a degree of 2.

The other equation is a linear and has a degree of 1.

Where is $f(x) = g(x)$? Justify your answer algebraically.

$-x^2 + 10x = -x + 14$

$+x - 14 \quad +x - 14$

$-x^2 + 11x - 14 = 0$

$\frac{-11 \pm \sqrt{(11)^2 - 4(-1)(-14)}}{2(-1)}$

$\frac{-11 \pm \sqrt{121 - 56}}{-2}$

$\frac{-11 \pm \sqrt{65}}{-2}$

$\frac{-11 \pm 8.1}{-2}$

$\frac{-11 + 8.1}{-2}$

$\frac{-2.9}{-2} = 1.45$

$\frac{-11 - 8.1}{-2} = \frac{-19.1}{2}$

9.55

Does your algebraic answer support with your graphic solution?

Yes

Let's Practice!

1. Consider the following system.

$$f(x) = 3$$
$$g(x) = x^2 - 7x + 11$$

$\frac{7}{2(1)} = \frac{7}{2} = 3.5$

$(3.5)^2 - 7(3.5) + 11$

$12.25 - 24.5 + 11 = -1.25$

Where is $f(x) = g(x)$? Justify your answer algebraically.

$x^2 - 7x + 11 = 3$

$x^2 - 7x + 8 = 0$

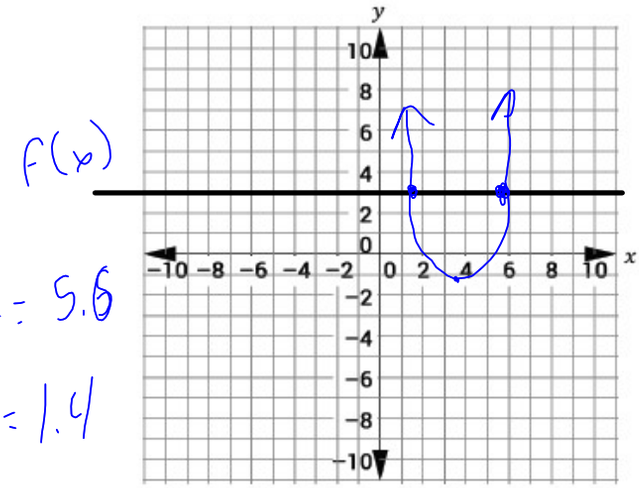
$\frac{7 \pm \sqrt{(-7)^2 - 4(1)(8)}}{2(1)}$

$\frac{7 \pm \sqrt{49 - 32}}{2} = \frac{7 \pm \sqrt{17}}{2}$

$\frac{7 + 4.1}{2} = 5.6$

$\frac{7 - 4.1}{2} = 1.4$

Graph the system.



Try It!

2. Consider the following system.

$$\begin{cases} f(x) = x^2 - 2x - 5 \\ g(x) = -2x^2 + 5x + 1 \end{cases}$$

$f(x) = \frac{2}{2(1)} = 1 \quad (1)^2 - 2 - 5 = -6$

$g(x) = \frac{-5}{2(-2)} = \frac{-5}{-4} = 1.25 \quad -2(1.25)^2 + 5(1.25) + 1$

$-3.1 + 6.25 + 1 = 4.2$

Where is $f(x) = g(x)$? Justify your answer algebraically. $(3)^2 - 2(3) - 5 = -2$

$x^2 - 2x - 5 = -2x^2 + 5x + 1$

$3x^2 - 5x - 1$

$3x^2 - 7x - 6 = 0$

$\frac{7 \pm \sqrt{(7)^2 - 4(3)(-6)}}{2(3)}$

$\frac{7 \pm \sqrt{49 + 72}}{6} = \frac{7 \pm \sqrt{121}}{6}$

$\frac{7+11}{6} = 3$

$\frac{7-11}{6} = \frac{-2}{3}$

Graph the system.

$\left(-\frac{2}{3}\right)^2 - 2\left(\frac{-2}{3}\right) - 5 \quad \frac{4}{9} + \frac{4}{9} - 5 = \frac{4}{9} + \frac{12}{9} - \frac{45}{9}$

$= \frac{-29}{9}$

$= -3.2$