Section 6: Polynomial Functions
Section 6 – Topic 1
Classifying Polynomials and Closure Property

A polynomial is a finite sum of monomials.

Determine whether each of the following expressions is a polynomial. If the expression is not a polynomial, change the expression so that it is a polynomial.

- no fractions as an exponent
- no variables in the denominator

$3x^2 + 2y + 4$ yes

$8a^{\frac{1}{2}} + 2c$ no $8a^2 + 2c$

$5r + \frac{s}{t}$ no $5r + st$

$\frac{5a + 4b}{2}$ yes $\frac{5a}{2} + 2b$

$9x^{-3} + 2y + 7x^3$ no $9x^3 + 2y + 7x^3$

5 yes

We can classify polynomials by the number of terms.

Number of Terms	Example	Name of Polynomial
1	$3x^7$	monomial
2	$5m + 6n$	binomial
3	$8r^2 + 4s + 7t$	trinomial
4 ↑	$a + 2b + c^2 + 4$	polynomial

We can also classify polynomials by degree.

Degree	Example	Type of Polynomial
0	3	constant
1	$2x + 3z$	linear
2	$3x^2 + 2y^2$	quadratic
3	$6y^3$	cubic
4	$5a^4 + 3b^3$	quartic

degree # of terms

Let's Practice!

1. Describe two polynomial functions that we have seen so far.

linear

quadratic

2. Explain if exponential functions are polynomial functions or not.

2^x - no because the x is in the exponential position

Try It!

Select the word that correctly completes each of the following statements.

3. A monomial is
- ◉ always
- ○ sometimes
- ○ never

a polynomial.

4. A polynomial is
- ○ always
- ◉ sometimes
- ○ never

a monomial.

5. A quadratic function is
- ◉ always
- ○ sometimes
- ○ never

a polynomial function.

We can also apply the ***Closure Property*** to polynomials.

A set is said to be closed under a specific mathematical operation if the result that occurs when you perform the operation on any two members of the set is also a member of the set.

Determine whether each of the following statements are true or false. If a statement is false, write a counterexample.

Integers are closed under addition.

True

Odd numbers are closed under addition.

False $3+5=8$

Even numbers are closed under addition.

True

Negative numbers are closed under multiplication.

False $(-3)(-4)=12$

Odd numbers are closed under multiplication. True

When referring to the Closure Property, what do you think "polynomials form a system analogous to the integers" means?

Integers are closed under +, −, • but not ÷.
Same goes for polynomials

Let's Practice!

6. Determine whether each of the following statements is true or false. If the statement is false, write a counterexample.

   a. Polynomials are closed under addition.

   True

   b. Polynomials are closed under subtraction.

   True

Try it!

7. Determine whether each of the following statements is true or false. If the statement is false, write a counterexample.

   a. Polynomials are closed under multiplication.

   True

   b. Polynomials are closed under division.

   False $\frac{2x}{2x^2} = \frac{1}{x}$

BEAT THE TEST!

1. Two functions are given below.

$$f(x) = x^3 + 2x^2 - 3x + 4$$
$$g(x) = x^2$$

Candice solved $\frac{f(x)}{g(x)}$ as follows:

$$\frac{x^3 + 2x^2 - 3x + 4}{x^2}$$

$$\frac{x^3}{x^2} + \frac{2x^2}{x^2} - \frac{3x}{x^2} + \frac{4}{x^2}$$

$$x + 2 - \frac{3}{x} + \frac{4}{x^2}$$

Part A: Candice's work illustrates that polynomials are

o closed
● not closed

under

o addition.
● division.
o multiplication.
o subtraction.

Part B: Explain your answer from Part A.