Geometric Series

A geometric series is the sum of all the terms in a geometric sequence.

Formula:

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

Note: Be careful with the rate in word problems.

Evaluate each finite series for the specified number of terms.

1. $40 + 20 + 10 + \ldots; n = 10$

a_1 $r = \frac{1}{2}$ or $.5$

$$\frac{40\left(1-\left(\frac{1}{2}\right)^{10}\right)}{1-\frac{1}{2}}$$

$$\frac{40\left(\frac{1023}{1024}\right)}{\frac{1}{2}}$$

$$\frac{40}{1}\cdot\frac{1023}{1024}\cdot\frac{2}{1} = 79.92$$

3. $15 + 12 + 9.6 + \ldots; n = 40$

a_1 $r = \frac{12}{15} = 0.8$

$$\frac{15\left(1-(0.8)^{40}\right)}{1-0.8}$$

$$\frac{15(1-.0001329923)}{0.2}$$

$$\frac{15(.9998670077)}{0.2}$$

$$= 74.99$$

5. $0.2 + 0.02 + 0.002 + \ldots; n = 8$

$r = 0.1$

a_1

$\frac{0.02}{0.2} =$

$$\frac{0.2(1-(0.1)^8)}{1-0.1}$$

$$\frac{0.2(1-0.00000001)}{0.9}$$

$$\frac{0.2(.99999999)}{0.9} = 0.2$$

7. This month, your friend deposits $400 to save for a vacation. She plans to deposit 10% more each successive month for the next 11 months. How much will she have saved after the 12 deposits?

$a_1 = 400 \quad n = 12$

$10\% = 0.1 + 1$

$r = 1.1$

$a_1 = 400$

$a_2 = 440$

$a_3 = 484$

$$\frac{400(1-(1.1)^{12})}{1-1.1}$$

$$\frac{400(1-3.138428377)}{-0.1}$$

$$\frac{400(-2.138428377)}{-0.1}$$

$\$8553.71$

9. Suppose your business made a profit of $5500 the first year. If the profit increased 20% per year, find the total profit over the first 5 yr.

$a_1 = 5500 \quad n = 5 \quad r = 1.2$ 20% = .2

$$\frac{5500(1-(1.2)^5)}{1-1.2}$$

$$\frac{5500(1-2.48832)}{-0.2}$$

$$\frac{5500(-1.48832)}{-0.2} = \$40,928.80$$

Calculating Loan Payments and Amount of Interest:

You can use geometric series to determine monthly payments on a loan:

$$x = \frac{\mathrm{Pr}(1+r)^n}{(1+r)^n - 1}$$

x is the monthly payment

P is the principle amount

r is the **monthly** interest rate

n is the number of **payments**

11. A man borrows a loan of $1,000,000 for a house from a bank and likes to pay back in 10 years (120 monthly instalments), the first instalment being paid at the end of first month and compound interest being calculated at 6% per annum. Find the amount of each instalment.

$6\% = \frac{.06}{12} = .005$

$$\frac{1{,}000{,}000(.005)(1.005)^{120}}{(1.005)^{120}-1}$$

$$\frac{1{,}000{,}000(.005)(1.819396734)}{.819396734}$$

$= \$11{,}102.05$

13. You are looking at a mortgage for a $160,000 home. You want a 15 year mortgage, which has an APR of 4.264%. Calculate your payment, then find out home much money you paid in interest.

$4.264\% = \frac{.0426\,4}{12} = .00355$

$160{,}000$

$$\frac{160{,}000(.00355)(1.00355)^{180}}{(1.00355)^{180}-1}$$

$$\frac{160{,}000(.00355)(1.892442745)}{.892442745} = \$1204.46$$

$$1204.46(180) - 160{,}000 = \$56801.97$$

15. You are looking at a 2015 Ford F-150 that costs $15,411. You get approved for a 4.14% interest rate on a 5 year loan. Determine the monthly payment, then calculate the total amount you will pay in interest.

12(5)

$4.14\% = \frac{.0414}{12} = .00345$

$$\frac{15{,}411(.00345)(1.00345)^{60}}{(1.00345)^{60}-1}$$

$$\frac{15{,}411(.00345)(1.22\ 954\ 4461)}{.22\ 954\ 4461} = \$284.79$$

$284.79(60) - 15411 = \$1676.40$

17. You are looking at a 2015 Ford F-150 that costs $15,411, but you have poor credit (below 600). You get approved for a 15.72% interest rate on a 5 year loan. Determine the monthly payment, then calculate the total amount you will pay in interest.

$15.72\% = \frac{.1572}{12} = .0131$

$$\frac{15411(.0131)(1.0131)^{60}}{(1.0131)^{60}-1}$$

$$\frac{15411(.0131)(2.18342831)}{.18342831} \quad \$372.48$$

$372.48(60) - 15411 = \$6937.80$