Graphs of Exponential Functions

In Coordinate Algebra one of the functions studied was the exponential function. By definition an exponential function is $y = a^x$ where $a > 0$ and $a \neq 1$. An exponential function returns powers of a base number a. The input of the exponential function is the exponent and the output is the number obtained when the base number is raised to that exponent.

1. For each function given, represent the function as a table and then use these points to graph the function on graph paper.

   a. $y = 2^x$
   b. $y = 3^x$
   c. $y = 4^x$
   d. $y = 10^x$

2. What common characteristics of these functions do you see? In particular, determine the domain and range of the functions and any intercepts. Also describe any characteristics of their graphs such as increasing/decreasing, asymptotes, end-behavior, etc.

   How does the graph of the exponential function change as the base a changes?

3. The symbol e represents the irrational number 2.718281828…. Recall an irrational number is represented by a non-terminating, non-repeating decimal number. e is one of those important numbers in mathematics like π that keeps showing up in all kinds of places. $y = e^x$ is the natural exponential function.

   Use graphing technology to graph $y = 2^x$, $y = 3^x$, and $y = e^x$. How do their graphs compare? What do you notice about the graph of $y = e^x$ in relationship to the graphs of $y = 2^x$ and $y = 3^x$?

4. Use graphing technology to graph each function.
   a. $y = 2^{-x}$
   b. $y = 3^{-x}$
   c. $y = 4^{-x}$
   d. $y = 10^{-x}$

How do these graphs compare to those in part (1) above? Use what you know about transformations of functions to explain the relationship between the graphs of $y = 2^x$ and $y = 2^{-x}$.

Does the same relationship hold for $y = 3^x$ and $y = 3^{-x}$? For $y = 4^x$ and $y = 4^{-x}$? In general, what is the relationship between the graphs of $y = a^x$ and $y = a^{-x}$?

5. Graph $y = \left(\frac{1}{2}\right)^x$. Compare its graph to $y = 2^{-x}$. What do you observe?

   Use properties of exponents to explain the relationship between $\left(\frac{1}{2}\right)^x$ and 2^{-x}.

   Do your observations about the graphs of $y = \left(\frac{1}{2}\right)^x$ and $y = 2^{-x}$ now make sense?

6. Graph $y = 2^x + 3$. How does this graph compare to that of $y = 2^x$?

   Based on what you know about transformations of functions, describe in words how $y = 2^x + 3$ transforms the graph of the parent function $y = 2^x$.

   Discuss what you notice about the domain, range, intercepts, and asymptote of $y = 2^x + 3$.

7. Graph $y = 2^{x-5}$. How does this graph compare to that of $y = 2^x$?

   Based on what you know about transformations of functions, describe in words how $y = 2^{x-5}$ transforms the graph of the parent function $y = 2^x$.

   Discuss what you notice about the domain, range, intercepts, and asymptote of $y = 2^{x-5}$.

8. The exponential function $y = a^x$ is defined for all real numbers $a > 0$ and $a \neq 1$.
   a. Why do you think the function is not defined for bases that are negative real numbers? Often to determine why something cannot be true, it helps to see what would happen if it were true!! So…explore what would happen for negative values of a; for example, see what would happen if $a = -2$. Set up a table of values to see if you can determine a reasonable explanation for why the base is not allowed to be negative in an exponential function.

   b. Why do you think the function is not defined for a base of 0 or a base of 1? Explore the functions $y = a^x$ for $a = 0$ and $a = 1$. Can you offer a reasonable explanation for excluding values of 0 and 1 for the base of an exponential function?