Remember that in a function, every input value corresponds to exactly one output value.

Consider the table below that represents the conversion of temperatures from degrees Fahrenheit to degrees Celsius.

Degrees Fahrenheit (Input)	-49	-22	14	122	167	212
Degrees Celsius (Output)	-45	-30	-10	50	75	100

This table defines a function since every input value corresponds to exactly one output value.

This table defines a function since every input value corresponds to exactly one output value.

Notice that every output value corresponds to exactly one input value.

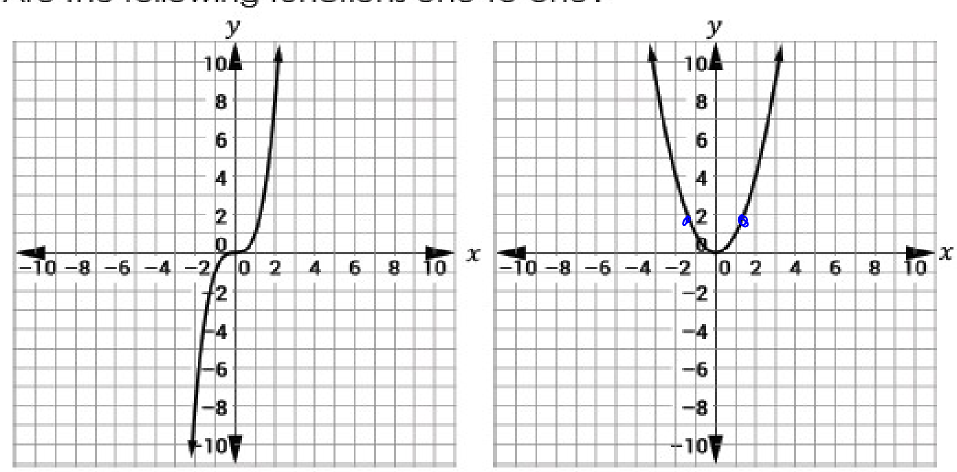
This is a special kind of function we call a(n) one to one function.

Are the following functions one-to-one?

$f: \{(-1,6), (0,5), (3,2), (7,10)\}$ Yes

$g: \{(-5,\underline{4}), (2,6), (3,5), (10,\underline{4})\}$ No

Are the following functions one-to-one?



Yes No

We can use the vertical line test to determine if a graph represents a function. What type of line test could we use to determine if the function is one-to-one?

horizontal

For every one-to-one function, we can find its ***inverse*** function. The output of the original function becomes the input of the inverse function.

The symbol f^{-1} is used to denote the inverse of the function ___f___.

We can find the inverse of a one-to-one function by switching the coordinates of the ordered pairs of the function.

Find the inverse of the following one-to-one function.

$f:\{(-1,3),(0,4),(2,-6),(3,6),(7,-8)\}$

$f^{-1}:\{(3,-1)(4,0)(-6,2)(6,3)(-8,7)\}$

When given a function $f(x)$, we can find the inverse, $f^{-1}(x)$, by interchanging x and y and solving for y.

Find the inverse of $f(x) = 5x + 2$.

$$y = 5x + 2$$

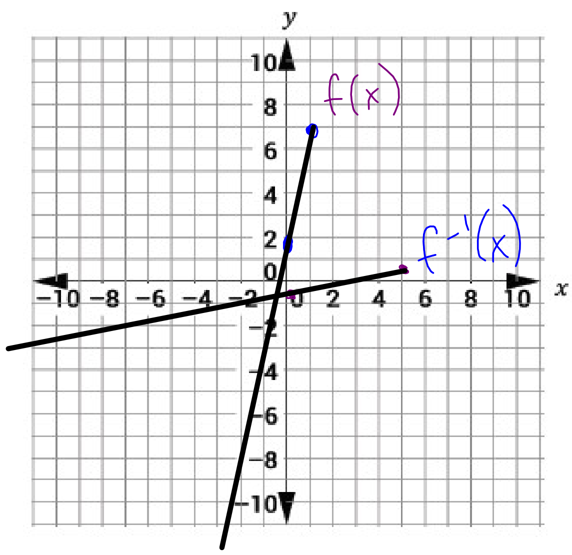
$$x = 5y \cancel{+2}$$

$$-2 \qquad \cancel{-2}$$

$$\frac{x}{5} - \frac{2}{5} = \frac{5y}{5}$$

$$\frac{x}{5} - \frac{2}{5} = y \rightarrow \boxed{f^{-1}(x) = \frac{x}{5} - \frac{2}{5}}$$

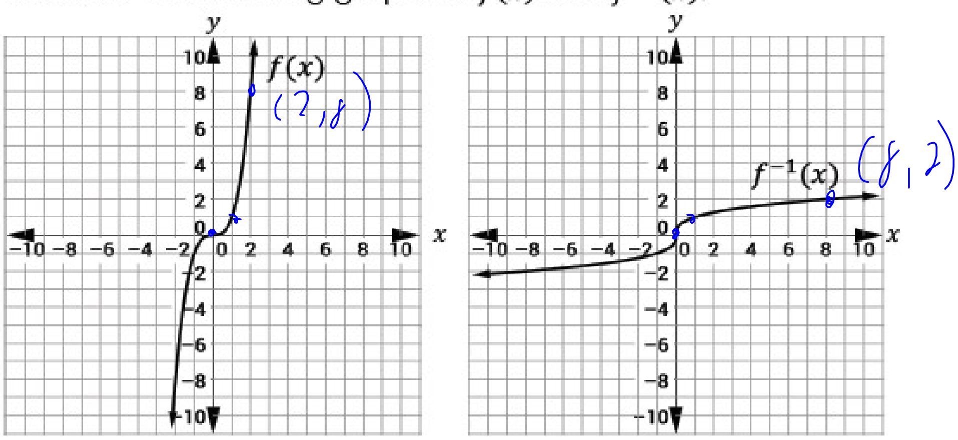
Graph the function and its inverse.



$$f(x) = 5x + 2$$

$$f^{-1}(x) = \frac{x}{5} - \frac{2}{5}$$

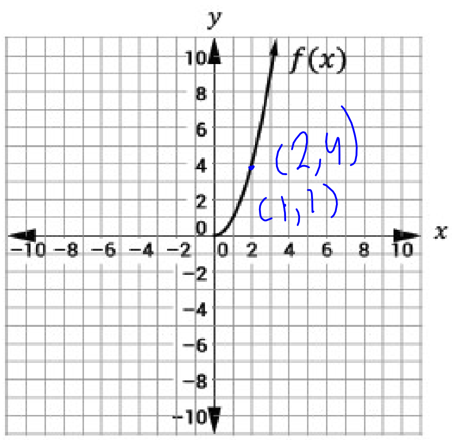
Consider the following graphs of $f(x)$ and $f^{-1}(x)$.

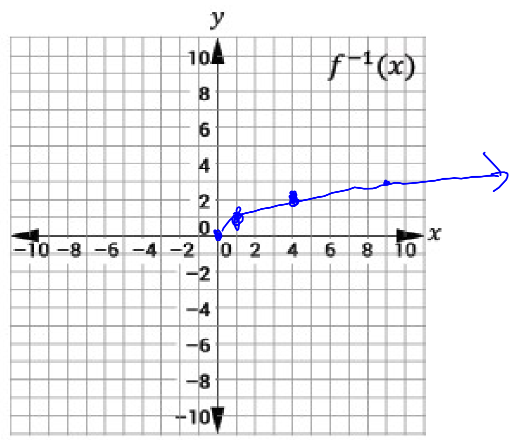


What do you notice about the graphs of $f(x)$ and $f^{-1}(x)$?

- both graphs go from neg to pos
- They are inverses and one-to-one functions.

Consider the following graph of $f(x)$. Graph $f^{-1}(x)$.





Try It!

1. Determine whether each function is a one-to-one function. If it is one-to-one, write the inverse function.

   a. $h: \{(11, 13), (4, 3), (3, 4), (8,8)\}$ yes

   $h^{-1}\{(13,11)(3,4)(4,3)(8,8)\}$

   b. $s: \{(2, 5), (3, -1), (7, 5), (6, 2)\}$

   No

2. Find the inverse of the following functions.

a. $f(x) = \frac{x-4}{7}$

$y = \frac{x-4}{7}$ $\quad (7)x = \frac{y-4}{\cancel{7}}\cancel{(7)} \rightarrow 7x = y - 4$

$\quad +4 \quad\quad +4$

$7x + 4 = y$ $\quad \boxed{f^{-1}(x) = 7x+4}$

b. $g(x) = \sqrt[3]{x+1}$

$y = \sqrt[3]{x+1}$

$x = \sqrt[3]{y+1}$

$x^3 = \sqrt[3]{y+1}^{\,3}$

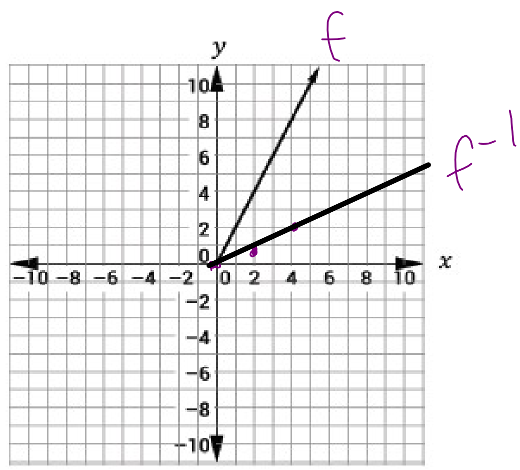
$x^3 = y + 1$

$-1 \quad\quad -1$

$x^3 - 1 = y$ $\quad \boxed{g^{-1}(x) = x^3 - 1}$

3. Graph the inverse of each function on the same coordinate plane.

a.



b.