

Inverse functions concluded

For every function $f(x)$, if the inverse of $f(x)$ is also a function, then the function $f(x)$ is an invertible function.

Determine if $f(x) = x^2$ is an invertible function. If not, restrict the domain so that $f(x)$ is an invertible function.

$$y = x^2$$
$$x = y^2$$
$$\sqrt{x} = y$$

$$f^{-1}(x) = \sqrt{x}$$
$$x \geq 0$$

We can also determine if functions are inverses of each other.

Consider the function $f(x) = 3x + 2$.

Find $f^{-1}(x)$.

$$y = 3x + 2$$

$$x = \frac{y + 2}{3}$$

$$\frac{x - 2}{3} = \frac{3y}{3}$$

$$\frac{x}{3} - \frac{2}{3} = y$$

$$f^{-1}(x) = \frac{x}{3} - \frac{2}{3}$$

Evaluate $f(f^{-1}(0))$.

$$f^{-1}(0) = \frac{0}{3} - \frac{2}{3}$$

$$f^{-1}(0) = -\frac{2}{3}$$

$$f\left(-\frac{2}{3}\right) = 3\left(-\frac{2}{3}\right) + 2$$

$$= -2 + 2$$
$$f\left(-\frac{2}{3}\right) = 0 \quad \checkmark$$

Evaluate $f^{-1}(f(6))$.

$$\begin{aligned} f(6) &= 3(6) + 2 \\ &= 18 + 2 \\ &= 20 \end{aligned}$$

$$\begin{aligned} f^{-1}(20) &= \frac{20}{3} - \frac{2}{3} \\ &= \frac{18}{3} \end{aligned}$$

$$f^{-1}(f(x)) = 6$$

Evaluate $f(f^{-1}(x))$.

$$f(x) = 3x + 2 \quad f^{-1}(x) = \left(\frac{x}{3} - \frac{2}{3} \right)$$

$$f(f^{-1}(x)) = 3 \left(\frac{x}{3} - \frac{2}{3} \right) + 2$$

$$(x - 2) + 2$$

$$f(f^{-1}(x)) = x$$

$$f(x) = 2x + 1$$

$$y = 2x + 1$$

$$x = 2y + 1$$

$$\frac{x-1}{2} = \frac{2y}{2}$$

$$\frac{x-1}{2} = y$$

$$f^{-1}(x) = \frac{x-1}{2}$$

yes

1. Is the function $f(x) = (x - 4)^2$ an invertible function? If not, restrict the domain so that $f(x)$ is an invertible function.

$$y = (x - 4)^2$$

$$x = (y - 4)^2$$

$$\sqrt{x} = \pm\sqrt{x}$$

$$\sqrt{x} + 4 = y - 4$$

$$\sqrt{x} + 4 = y$$

$$f^{-1}(x) = \sqrt{x} + 4$$

$$x \geq 0$$

2. If $f(x) = x^3 - 5$, show that $f^{-1}(x) = \sqrt[3]{x+5}$.

index

$$y = x^3 - 5$$

$$x = y^3 - 5$$

$$+5 \quad +5$$

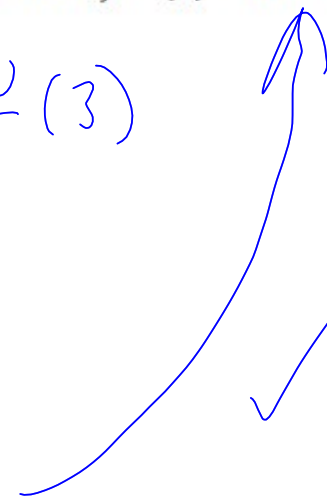
$$x + 5 = y^3$$

$$\sqrt[3]{x+5} = y$$

$$f^{-1}(x) = \sqrt[3]{x+5}$$

Try It!

3. If $f(x) = \frac{x-10}{3}$, show that $f^{-1}(x) = 3x + 10$.

$$\begin{aligned} (3) \quad y &= \frac{x-10}{3} \quad (3) \\ 3y &= x-10 \\ 3x &= y-10 \\ +10 \quad +10 & \\ 3x+10 &= y \end{aligned}$$


1. Two functions are given:

$$f(x) = 2x + 4$$

$$g(x) = \frac{1}{2}x - 2$$

Some of these steps are used in the composition of functions to determine if $f(x)$ and $g(x)$ are inverses.

A. $f\left(\frac{1}{2}x - 2\right)$	B. x	C. $2\left(\frac{1}{2}x - 2\right)$
D. $\frac{1}{2}(2x + 4)$	E. $x - 4$	F. $g(2x + 4)$
G. $x - 4 + 4$	H. $x + 2$	I. $x + 2 - 2$
J. x	K. $\frac{1}{2}(2x + 4) - 2$	L. $2\left(\frac{1}{2}x - 2\right) + 4$

Part A: Rearrange the steps in the correct order and write the steps in the correct spaces below.

$(f \circ g)(x) =$	<input type="text" value="A"/>	$g(f(x)) =$	<input type="text" value="F"/>
$=$	<input type="text" value="L"/>	$=$	<input type="text" value="K"/>
$=$	<input type="text" value="G"/>	$=$	<input type="text" value="I"/>
$=$	<input type="text" value="J"/>	$=$	<input type="text" value="B"/>

Part B: Which of the following statements is correct?

- Ⓐ $f(x)$ and $g(x)$ are not inverses of each other because the inverse of $f(x)$ is $\frac{1}{4}x - 2$.
- Ⓑ $f(x)$ and $g(x)$ are not inverses of each other because the inverse of $g(x)$ is $2x + 2$.
- Ⓒ $f(x)$ and $g(x)$ are not inverses of each other, but they are perpendicular to each other.
- Ⓓ $f(x)$ and $g(x)$ are inverses of each other.