Inverse functions concluded

For every function $f(x)$, if the inverse of $f(x)$ is also a function, then the function $f(x)$ is an invertable function.

Determine if $f(x) = x^2$ is an invertible function. If not, restrict the domain so that $f(x)$ is an invertible function.

$y = x^2$

$x = y^2$

$\sqrt{x} = y$

$f^{-1}(x) = \sqrt{x}$

$x \geq 0$

We can also determine if functions are inverses of each other.

Consider the function $f(x) = 3x + 2$.

Find $f^{-1}(x)$.

$y = 3x + 2$

$x = 3y + 2$
$-2 \qquad -2$

$\frac{x}{3} - \frac{2}{3} = \frac{3y}{3}$

$\frac{x}{3} - \frac{2}{3} = y$

$f^{-1}(x) = \frac{x}{3} - \frac{2}{3}$

Evaluate $f(f^{-1}(0))$.

$f^{-1}(0) = \frac{0}{3} - \frac{2}{3}$

$f^{-1}(0) = -\frac{2}{3}$

$f\left(\frac{2}{3}\right) = 3\left(-\frac{2}{3}\right) + 2$

$= -2 + 2$

$f\left(\frac{2}{3}\right) = 0$ ✓

Evaluate $f^{-1}(f(6))$.

$f(6) = 3(6) + 2$
$= 18 + 2$
$= 20$

$f^{-1}(20) = \frac{20}{3} - \frac{2}{3}$
$= \frac{18}{3}$

$f^{-1}(f(x)) = 6$

Evaluate $f(f^{-1}(x))$.

$f(x) = 3x + 2$ $\quad f^{-1}(x) = \frac{x}{3} - \frac{2}{3}$

$f(f^{-1}(x)) = \cancel{3}\left(\frac{x}{\cancel{3}} - \frac{2}{\cancel{3}}\right) + 2$

$(x - \cancel{2}) + \cancel{2}$

$f(f^{-1}(x)) = x$

$$f(x) = 2x + 1 \qquad f^{-1}(x) = \frac{x-1}{2}$$

$$y = 2x + 1$$

$$x = 2y + 1$$
$$\;-1 \qquad -1$$

$$\frac{x-1}{2} = \frac{\cancel{2}y}{\cancel{2}}$$

$$\frac{x-1}{2} = y$$

yes

1. Is the function $f(x) = (x-4)^2$ an invertible function? If not, restrict the domain so that $f(x)$ is an invertible function.

$y = (x-4)^2$

$\sqrt{x} = \sqrt[2]{\cancel{x}}$

$x = (y-4)^2$

$\sqrt{x} = y - 4$

$+4 \qquad +4$

$\sqrt{x} + 4 = y$

$f^{-1}(x) = \sqrt{x} + 4$

$x \geq 0$

2. If $f(x) = x^3 - 5$, show that $f^{-1}(x) = \sqrt[3]{x+5}$.

index $\sqrt{}$

$y = x^3 - 5$

$x = y^3 - 5$

$+5 \quad +5$

$x + 5 = y^3$

$\sqrt[3]{x+5} = y$

$f^{-1}(x) = \sqrt[3]{x+5}$

Try It!

3. If $f(x) = \frac{x-10}{3}$, show that $f^{-1}(x) = 3x + 10$.

$(3)\ y = \frac{x-10}{3}\ (3)$

$3y = x - 10$

$3x = y - 10$
$+10 \quad\quad +10$

$3x + 10 = y$

✓

1. Two functions are given:

$f(x) = 2x + 4$ $\qquad$ $g(x) = \frac{1}{2}x - 2$

Some of these steps are used in the composition of functions to determine if $f(x)$ and $g(x)$ are inverses.

A. $f\left(\frac{1}{2}x - 2\right)$	**B.** x	**C.** $2\left(\frac{1}{2}x - 2\right)$
D. $\frac{1}{2}(2x + 4)$	**E.** $x - 4$	**F.** $g(2x + 4)$
G. $x - 4 + 4$	**H.** $x + 2$	**I.** $x + 2 - 2$
J. x	**K.** $\frac{1}{2}(2x + 4) - 2$	**L.** $2\left(\frac{1}{2}x - 2\right) + 4$

Part A: Rearrange the steps in the correct order and write the steps in the correct spaces below.

$(f \circ g)(x) =$	A	$g(f(x)) =$	F
$=$	L	$=$	K
$=$	G	$=$	I
$=$	J	$=$	B

Part B: Which of the following statements is correct?

- Ⓐ $f(x)$ and $g(x)$ are not inverses of each other because the inverse of $f(x)$ is $\frac{1}{4}x - 2$.
- Ⓑ $f(x)$ and $g(x)$ are not inverses of each other because the inverse of $g(x)$ is $2x + 2$.
- Ⓒ $f(x)$ and $g(x)$ are not inverses of each other, but they are perpendicular to each other.
- Ⓓ $f(x)$ and $g(x)$ are inverses of each other.