Inverse Variation: $y = \frac{k}{x}$, $k = xy$

Combined Variations

1. z varies jointly with x and y $z = kxy$
2. z varies jointly with x and y and inversely with w $z = \frac{kxy}{w}$
3. z varies directly with x and inversely with the product of wy $z = \frac{kx}{wy}$

Inverse $k = yx$ Direct $k = \frac{y}{x}$

Is the relationship between the values in each table a *direct variation*, an *inverse variation*, or *neither*? Write equations to model the direct and inverse variations.

1) $y = \frac{k}{x}$

x	2	4	5	20
y	10	5	4	1

$k = 20$

Inverse

$y = \frac{20}{x}$

(2) $y = kx$

x	1	2	5	7
y	6	12	30	42

$k = 6$

Direct

$y = 6x$

(3) 2 $\frac{8}{3}$

x	1	3	7	10
y	2	8	20	29

Neither

$k = xy$ $y = k/x$

Suppose that x and y vary inversely. Write a function that models each inverse variation. Graph the function and find y when $x = 10$.

4) $x = 7$ when $y = 2$

$k = 2(7)$

$k = 14$

$y = \frac{14}{x}$ $\frac{14}{10} = \frac{7}{5}$

$(7, 2)$

$(10, \frac{7}{5})$

5) $x = \frac{1}{3}$ when $y = \frac{9}{10}$

$k = \frac{1}{3}\left(\frac{9}{10}\right) = \frac{3}{10}$

$y = \frac{\frac{3}{10}}{x} = \frac{3}{10} \div \frac{x}{1}$

$\frac{3}{10} \cdot \frac{1}{x}$

$y = \frac{3}{10x} = \frac{3}{10(10)} = \frac{3}{100}$

$(\frac{1}{3}, \frac{9}{10})$ $(10, \frac{3}{100})$

Write the function that models each variation. Find z when $x = 6$ and $y = 4$.

6) z varies jointly with x and y. When $x = 7$ and $y = 2$, $z = 28$.

directly

$z = kxy$

$28 = k(7)(2)$

$k = 2$

$z = 2xy$

$z = 2(6)(4)$

$z = 48$

7) z varies directly with x and inversely with the cube of y. When $x = 8$ and $y = 2$, $z = 3$.

$z = \frac{kx}{y^3}$

$3 = \frac{k(8)}{2^3}$

$k = 3$

$z = \frac{3x}{y^3}$

$z = \frac{3(6)}{4^3} = \frac{18}{64} = \frac{9}{32}$

Each ordered pair is from an inverse variation. Find the constant of variation.

8) $(10, 5)$

$k = xy$

$k = 50$

9) $(-13, 22)$

$k = -286$

10) $\left(\frac{1}{3}, \frac{6}{7}\right)$

$k = \frac{6}{21} = \frac{2}{7}$

Each pair of values is from an inverse variation. Find the missing value.

$k = xy$

11) $(2, 4), (6, y)$

$8 = 6y$

$\frac{8}{6} = y$

$\frac{4}{3} = y$

12) $(1.2, 4.5), (2.7, y)$

$(1.2)(4.5) = 2.7y$

$5.4 = 2.7y$

$y = 2$