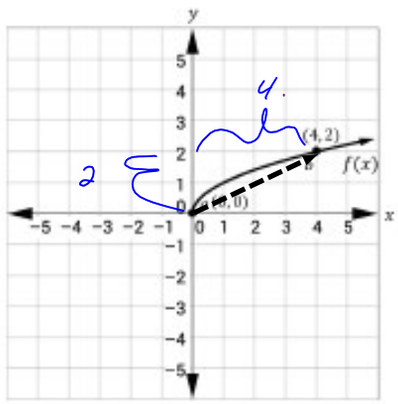
Section 3 -[redacted]

Average Rate of Change Over an Interval

Consider the following graph of the square root function $f(x) = \sqrt{x}$.



Draw a line connecting a and b.

Determine the slope of the line between the interval $[a, b]$.

$\frac{2}{4} = \frac{1}{2}$

For every two points x_1 and x_2, where $x_1 \neq x_2$, (x_1, y_1) and (x_2, y_2) form a straight line and create a _____ _____.

To determine the average rate of change for any function $f(x)$ over an interval, we can use two points $(x_1, \underline{y_1})$ and $(x_2, \underline{y_2})$ that lie on that interval.

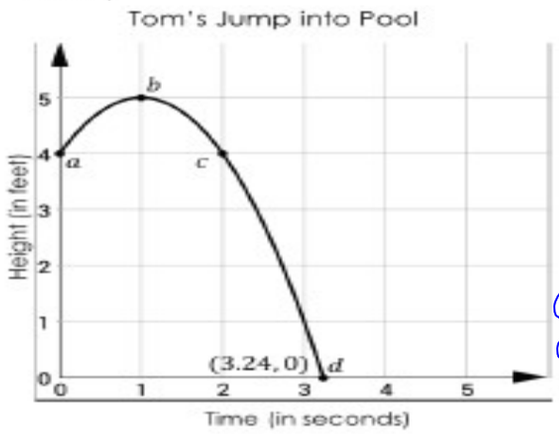
The process to find the slope of a linear function is:

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

We can also use the slope formula to find the average rate of change over an interval $[a, b]$, where $x_1 = a$ and $x_2 = b$.

Let's Practice!

1. Tom is jumping off the diving board at the Tony Dapolito Pool. His height is modeled by the quadratic function $h(t) = -t^2 + 2t + 4$, where $h(t)$ represents height above water (in feet), and t represents time after jumping (in seconds).



a. Determine the average rate of change for the following intervals.

$[a, b]$ $\frac{1}{1}$

$[b, c]$ $-\frac{1}{1}$

$[c, d]$ $-\frac{4}{1}$

b. Compare Tom's average rate of change over the interval $[a, b]$ with his average rate of change over the interval $[b, c]$. What does this represent in real life?

[a,b] - up 1 foot in 1 second
[b,c] - down 1 foot in 1 second

Try It!

2. Consider the table for the exponential function, $p(x) = 3^x$, shown below.

M (0, 1)
N (1, 3)
R (2, 9)
T (3, 27)

Point	x	$p(x)$
M	0	1
N	1	3
R	2	9
T	3	27

a. Determine the average rate of change over the interval $[N, T]$.

$$\frac{27-3}{3-1} = \frac{24}{2} = \frac{12}{1}$$

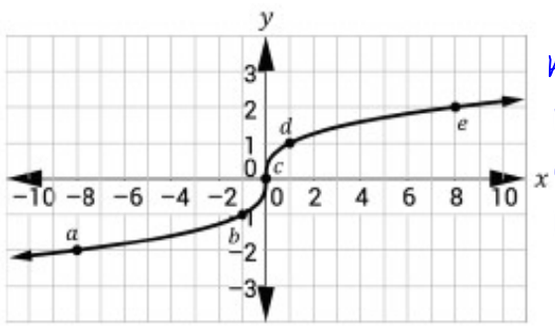
b. Compare the average rate of change over the interval $[M, N]$ with the average rate of change over the interval $[R, T]$.

$$MN = \frac{3-1}{1-0} = \frac{2}{1} \quad \overline{RT} \quad \frac{27-9}{3-2} = \frac{16}{1}$$

The slope of RT is 8 times bigger than MN

3. Determine the intervals that have the same average rate of change in the graph $f(x) = \sqrt[3]{x}$ below.

$\frac{y_2 - y_1}{x_2 - x_1}$



A(-8, -2)
B(-1, -1)
C(0, 0)
D(1, 1)
E(8, 2)

$\overline{AB} = \frac{-1-(-2)}{-1-(-8)} = \frac{1}{7}$

$\overline{BC}$ $\frac{0-(-1)}{0-(-1)} = \frac{1}{1}$

$\overline{CD}$ $\frac{1-0}{1-0} = \frac{1}{1}$

$\overline{DE}$ $\frac{2-1}{8-1} = \frac{1}{7}$

AB + DE

BC, + CD

BEAT THE TEST!

1. Suppose that the cost of producing r radios is defined by $c(r) = 300 + 15r - 0.3r^2$. Determine which of the following intervals has the greatest average rate of change for the cost to produce a radio.

   Ⓐ Between 20 and 25 radios. (20, 480)(25, 437.5)
   Ⓑ Between 60 and 65 radios. (60, 120)(65, 7.5)
   Ⓒ Between 5 and 10 radios. (5, 367.5)(10, 420)
   Ⓓ Between 30 and 35 radios. (30, 480)(35, 457.5)

$300 + 15(20) - .3(20)^2$
$300 + 300 - 120$
$600 - 120 = 480$

$300 + 15(25) - .3(25)^2$
$300 + 375 - 187.5 =$
$625 - 187.5 = 437.5$

Ⓐ $\frac{-42.5}{5}$

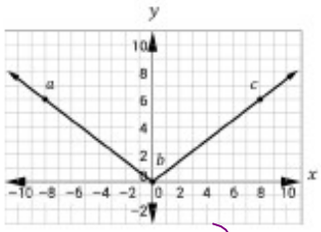
Ⓑ $\frac{-112.5}{5}$

Ⓒ $\frac{53.5}{5}$

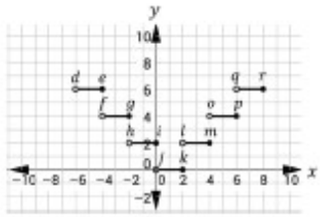
Ⓓ $\frac{-22.5}{5}$

2. Consider the absolute value function $f(x)$ and the step function $g(x)$ in the graphs below.

$f(x)$



$g(x)$



Which of the following is true about the rate of change of the graphs?

- The average rate of change for $f(x)$ over the interval $[b, c]$ is greater than the average rate of change for $g(x)$ over the interval $[j, k]$.
- The average rate of change for $f(x)$ over the interval $[a, c]$ is greater than the average rate of change for $g(x)$ over the interval $[d, r]$.
- The average rate of change for $f(x)$ over the interval $[a, b]$ is $-\frac{1}{2}$.
- The average rate of change for $g(x)$ over the interval $[d, j]$ is $-\frac{1}{2}$.

B(0,0)
C(8,6)

$\frac{6-0}{8-0} = \frac{6}{8} = \frac{3}{4}$

j(0,0)
k(2,0) > slope of 0