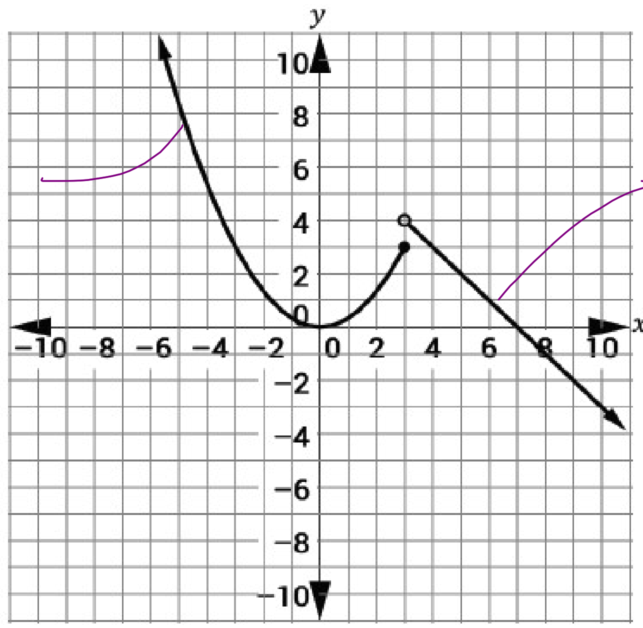
Section 3 – Topic 1
Introduction to Piecewise-Defined Functions – Part 1

Consider the following piecewise-defined function, and make observations about its graph.

parabola (quadratic)

$\{x \mid x \le 3\}$

such as

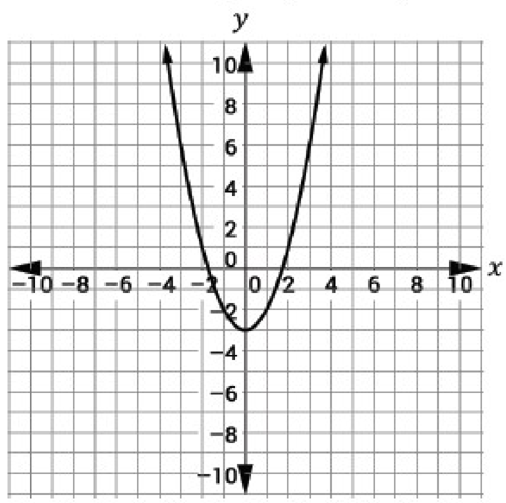


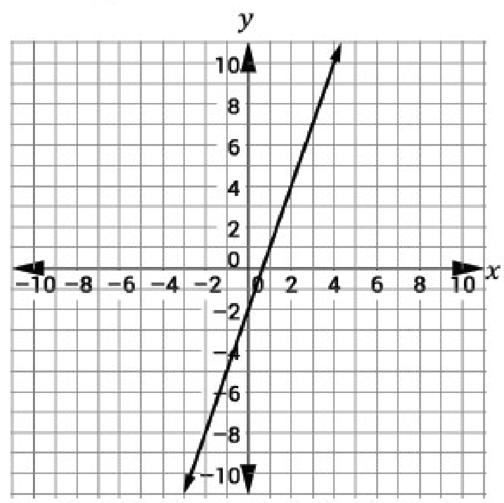
linear

$\{x \mid x > 3\}$

A piecewise-defined function is made up of defined pieces based on different rules for the domain.

Consider the graphs of $y = x^2 - 3$ and $y = 3x - 2$.





Consider the following piecewise-defined function.

[ ] included

( ) not included

$$f(x) = \begin{cases} x^2 - 3, & x \leq 0 \\ 3x - 2, & x > 0 \end{cases}$$

The defined domain of the function $x^2 - 3$ is $(-\infty, 0]$.

The defined domain of the function $3x - 2$ is $(0, \infty)$.

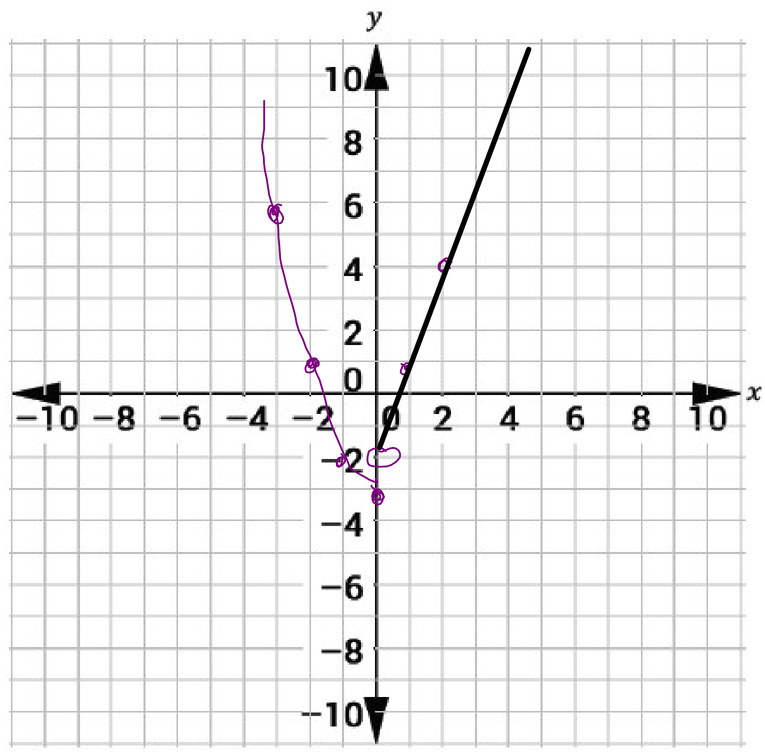
This means that each of these functions is graphed only for the x-values identified in the defined domain.

Use the graphs of $y = x^2 - 3$ for $x \le 0$ and $y = 3x - 2$ for $x > 0$ to graph the piecewise-defined function.

$x \le 0$

$x^2 - 3$

x	y
0	-3
-1	-2
-2	1
-3	6



$x > 0$

$3x - 2$

x	y
0	-2
1	1
2	4

Let's note some features of the graph.

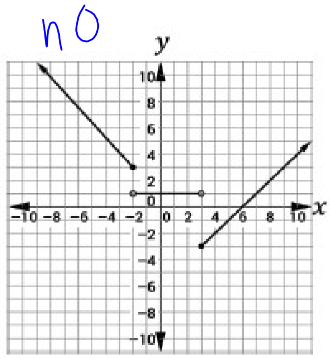
- The domain of the piecewise graph can be represented with intervals. We can define interval A as $x \leq 0$ and interval B as $x > 0$.
- Over interval A, the function is $\{x \mid x \leq 0\}$, and it is $\{x \mid x > 0\}$ over interval B.
- The graph is nonlinear (curved) when the domain is $(-\infty, 0]$.
- The graph is linear when the domain is $(0, \infty)$.

- There is one closed circle on the graph, which means that the particular value, $x = 0$, is included in that piece of the function. This illustrates the inclusion of zero in $x \leq 0$.

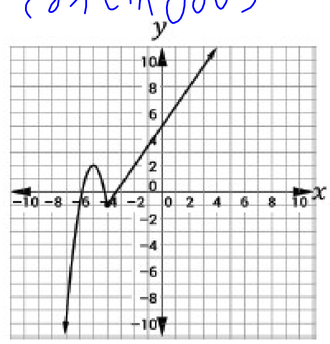
- There is one open circle on the graph, which means that the particular value, $x = 0$, is not included in that piece of the function. This illustrates the constraint that $x > 0$.

Consider the following graphs of piecewise functions. Which ones do you think would be considered continuous?

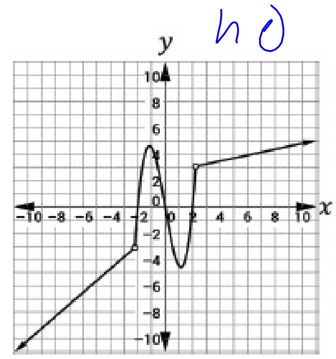
no



continuous

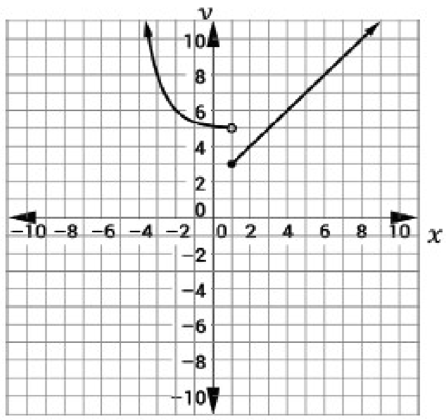


no



Try It!

1. Consider the graph below of a piecewise function.



a. Over what interval(s) is the function increasing?

$\{x \mid x \geq 1\}$

b. Over what interval(s) is the function decreasing?

$\{x \mid x < 1\}$

c. What is the domain of the nonlinear piece?

$(-\infty, 1)$

d. What is the domain of the linear piece?

$(1, \infty)$

e. What is the range of the function?

$[3, \infty)$

f. Does the graph represent a function? Explain how you know. yes, each x value has exactly one y value.

g. Is this piecewise function continuous? no