Section 4 – Topic 2
Solving Quadratic Equations by Factoring

To solve a quadratic equation by factoring. Write the quadratic in standard form, $ax^2 + bx + c = 0$ and factor out the greatest common factor, if possible.

There are two ways to factor.

- Use the ~~area model~~ master product method to factor the trinomial.
- Use factoring by grouping.

Then, use the zero product property to find the solutions.

Let's Practice!

1. Consider the equation $2c^2 - 9 = 3c$.

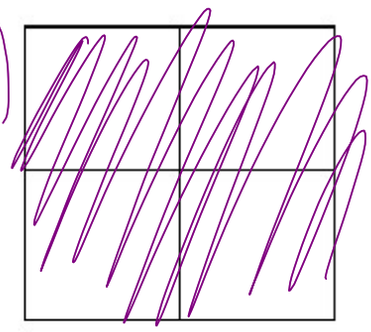
$2c^2 - 3c - 9 = 0$

$-3c \quad -3c$

$a = 2 \quad b = -3 \quad c = -9$

a. Use the area model to factor.

$(c + \frac{3}{2})(c - \frac{6}{2})$



$(2c+3)(c-3)$

product	sum
a·c -18	-3 b
$2(9)$	
$3(6)$	$3-6$
$1(18)$	

÷ by a

$2c^2 - 3c - 9$

b. Factor by grouping.

$2c^2 + 3c - 6c - 9$

$(2c^2 + 3c)(-6c - 9)$

$c(2c+3) - 3(2c+3)$

$(2c+3)(c-3)$

Try it!

2. Solve the quadratic equation below using factoring by grouping. Identify the property used in each step to solve the equation.

$$11x + 2 = -5x^2$$

$+5x^2 \quad +5x^2$

$5x^2 + 11x + 2 = 0$

$a = 5 \quad b = 11 \quad c = 2$

$\left(x + \frac{10}{5}\right)\left(x + \frac{1}{5}\right)$

$(x+2)(5x+1)$

a·c	b
10	11
1(10)	1+10
2(5)	

Let's investigate factoring by substitution.

Perfect square

Consider the quadratic equation $36x^2 + 60x + 21 = 0$.

u^2 10 $6x$ 21

Let $u^2 = 36x^2$ in the trinomial.

$\sqrt{u^2} = u$ $\sqrt{36x^2} = 6x$ $10u$

What does $60x$ equal in terms of u?

$u = 6x$

Rewrite the quadratic equation in terms of u. Factor and solve for u.

$u^2 + 10u + 21 = 0$

$a = 1$ $b = 10$ $c = 21$

$(u+3)(u+7) = 0$

21	10
3(7)	3+7

Solve for x.

$u + 3 = 0$ $u + 7 = 0$

$u = -3$ $u = -7$

$\frac{-3 = 6x}{6}$ $\frac{-7 = 6x}{6}$

$-\frac{1}{2} = x$ $-\frac{7}{6} = x$

Let's Practice!

3. Use factoring by substitution to solve the equation below.

$u^2 = 25x^2$

$u = 5x$

$25x^2 + 15x - 18 = 0$

$3 \quad 5x$

$\downarrow \quad \vee$

$3u$

$-18 \mid 3$

$(3)(6) \mid 6 - 3$

$u^2 + 3u - 18 = 0$

$(u+6)(u-3) = 0$

$u = -6 \quad u = 3$

$u = 5x$

$-6 = 5x \qquad 3 = 5x$

$\frac{-6}{5} = x \qquad \frac{3}{5} = x$

Try It!

4. Use factoring by substitution to solve for x in the equation below.

$$9x^4 - 18x^2 + 8 = 0$$

$u^2 = 9x^4$

$u = 3x^2$

$8 \mid -6$

$2(4) \mid -4-2$

$u^2 - 6u + 8 = 0$

$(u-4)(u-2) = 0$

$u = 4, 2$

$u - 4 = 0$

$u - 2 = 0$

$\frac{4 = 3x^2}{3}$ $\quad$ $\frac{2 = 3x^2}{3}$

$\frac{4}{3} = x^2$ $\quad$ $\frac{2}{3} = x^2$

$\pm\sqrt{\frac{4}{3}} = x$ $\quad$ $\pm\sqrt{\frac{2}{3}} = x$

$\pm\sqrt{\frac{4}{3}} = \pm\frac{2\sqrt{4}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{9}} = \pm\frac{2\sqrt{3}}{3}$

$+\frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$

BEAT THE TEST!

$P = 2l + 2w$

1. A rectangle has an area of $4x^2 - 8x + 3$ square units. Which of the following could represent the perimeter of the rectangle in terms of x?

   Ⓐ $2x - 1$
   Ⓑ $2x - 3$
   Ⓒ $4x - 4$
   Ⓓ $8x - 8$

a = 4

a · c = 12 | −8

1 (12)
3 (4)
2 (6) | −2 −6

$\frac{-2}{4}$ $\frac{-6}{4}$

$-\frac{1}{2}$ $-\frac{3}{2}$

$(x - \frac{1}{2})(x - \frac{3}{2})$

$(2x-1)(2x-3)$

$2(2x-1) + 2(2x-3)$

$4x - 2 + 4x - 6 = 8x$

$u = 3 \quad u = -1$

2. Giovanni factored $16x^2 - 8x - 3 = 0$ as $(u - 3)(u + 1) = 0$.

$u^2 = 16x^2$

$u = 4x$

Which of the following are solution(s) to $16x^2 - 8x - 3 = 0$? Select all that apply.

- ☐ -1
- ☒ $-\frac{1}{4}$
- ☐ 0
- ☒ $\frac{3}{4}$
- ☐ 3

$\frac{3 = 4x}{4} \qquad \frac{-1 = 4x}{4}$

$\frac{3}{4} = x \qquad \frac{-1}{4} = x$