Section 5 – Topic 13
Classifying Quadratic Functions and Finding Inverses

$f(x) = 3x^2 + 4x + 7$ $f(-x) = 3x^2 + 4x - 7$

How can we determine that a function is even?

$f(-x) = f(x)$ symmetric about the y-axis

How can we determine that a function is odd?

$f(x) = 4x + 7$
$-f(x) = -4x - 7$

$f(-x) = -f(x)$ symmetric about the origin

Let's Practice!

1. Quadratic functions are
   - ○ always
   - ◉ sometimes
   - ○ never

   even.

2. Quadratic functions are
   - ○ always
   - ○ sometimes
   - ◉ never

   odd.

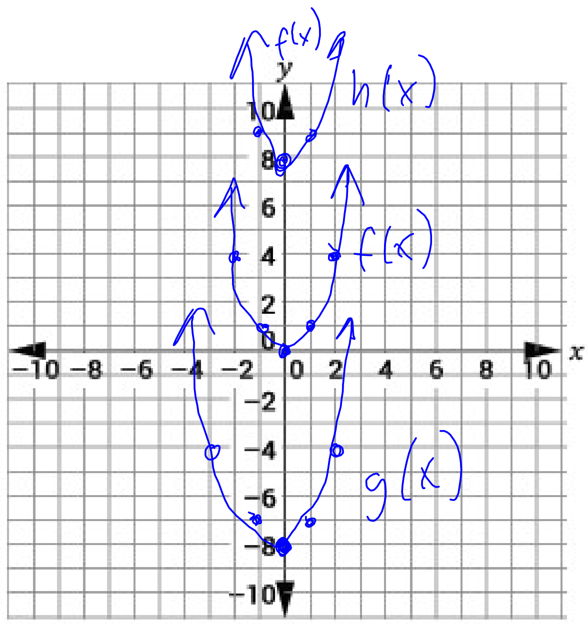
Try It! an even quadratic function has its vertex on the y axis

3. Sketch the graphs of three even quadratic functions; one with two solutions, one with one solution, and one with no solutions.

$f(x) = x^2$

$g(x) = x^2 - 8$

$h(x)$ $x^2 + 8$



# of solutions

f(x) = one

g(x) = two

h(x) = none

4. Give algebraic representations of three even quadratic functions; one with two solutions, one with one solution, and one with no solutions.

How to determine the inverse of a function:

Step 1: Write function notation $f(x)$ as y.
Step 2: switch the variables y and x.
Step 3: solve the equation for y.
Step 4: Write in function notation $f^{-1}(x)$.

$f(x) = 4x-7$
$y = 4x-7$
$x = 4y-7$
$x+7 = 4y$
$\frac{x+7}{4} = y$
$f^{-1}(x) = \frac{x+7}{4}$

There are two ways to determine if two functions are inverses:

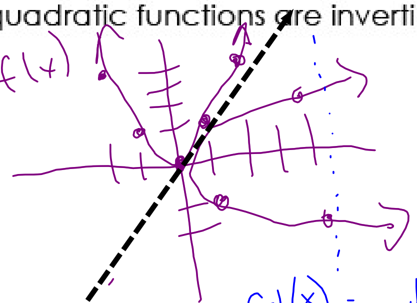
Algebraically: Functions $f(x)$ and $g(x)$ are inverses if $f(g(x)) = g(f(x))$.

Graphically: Functions $f(x)$ and $g(x)$ are inverses if they are reflections over the line $y = x$.

Determine if quadratic functions are invertible. Justify your answer.

$f(x)$
(0,0) (2,4)
(1,1) (-2,4)
(-1,1)
$f(x) = x^2$

f(x)



$f^{-1}(x)$
(0,0) (4,2)
(1,1) (4,-2)
(1,-1)

$f^{-1}(x)$ = not a function

f(x) & $f^{-1}(x)$ are not invertable

Let's Practice!

5. Consider the quadratic function $f(x) = (x + 3)^2 + 2$.

vertex $(-3, 2)$

a. Restrict the domain so that $f(x)$ is invertible.

$f(x) = (x+3)^2 + 2 \quad x \leq -3 \rightarrow f^{-1}(x) = -3 - \sqrt{x-2}$

$f(x) = (x+3)^2 + 2 \quad x \geq -3 \rightarrow f^{-1}(x) = -3 + \sqrt{x-2}$

b. Find the inverse for each domain.

$\boxed{f^{-1}(x) = -3 \pm \sqrt{x-2}}$

$y = (x+3)^2 + 2$

$x = (y+3)^2 + 2$

$x - 2 = (y+3)^2$

$\pm\sqrt{x-2} = y + 3$

$-3 \qquad -3$

$-3 \pm \sqrt{x-2} = y$

$f(x) = (x+3)^2 + 2$

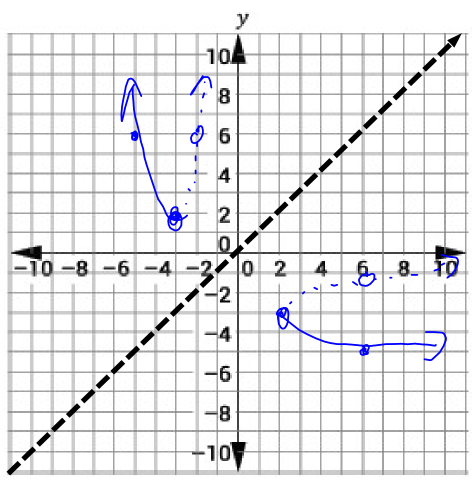
$x = -1,\ f(x) = 6$

$x = -3\ \ f(x) = 2$

$x = -5\ \ f(x) = 6$

c. Sketch the graph of the quadratic function with the restricted domains and its inverse.

$y = x$



$x = 2,\ f^{-1}(x) = -3$

$x = 6,\ f^{-1}(x) = -1$

$x = 6,\ f^{-1}(x) = -5$

Try It!

6. Consider the functions $g(x) = x^2 - 8x + 17$ for $x \geq 4$ and $h(x) = \sqrt{x-1} + 4$.

a. Prove that $h(x)$ and $g(x)$ are inverses algebraically.

$h(g(x)) = \sqrt{(x^2-8x+17)-1} + 4$

$\sqrt{x^2} = x$

$\sqrt{16} = 4$

$\sqrt{(x^2-8x+16)} + 4$

$\sqrt{(x-4)^2} + 4$

$h(g(x)) = x-4+4$

$h(g(x)) = x$

b. Show that $h(x)$ and $g(x)$ are inverses by graphing.

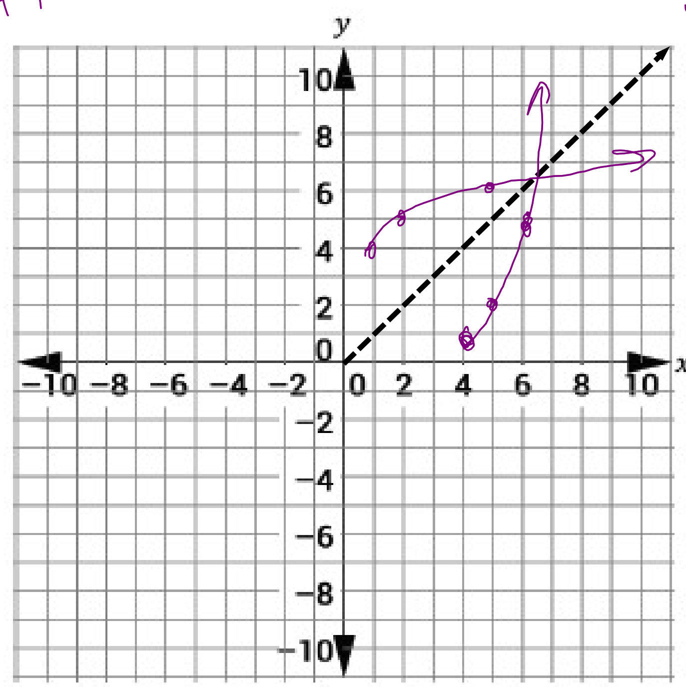
$x \geq 4$

$g(x) = \sqrt{x-1} + 4$

$(1,4)$

$(2,5)$

$(5,6)$



$g(x) = x^2 - 8x + 17$

$\frac{-b}{2a} = \frac{8}{2(1)} = 4$ $(4,1)$

$(4)^2 - 8(4) + 17$

$16 - 32 + 17 = 1$

$(5)^2 - 8(5) + 17$ $(5,2)$

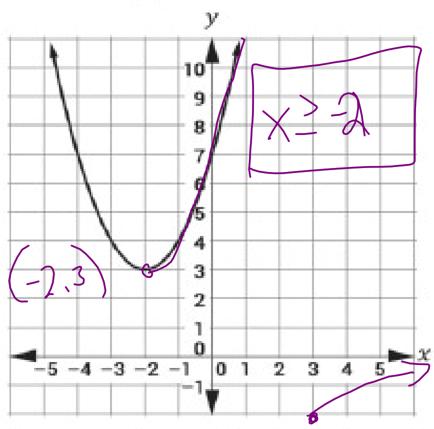
$25 - 40 + 17 = 2$

$(6)^2 - 8(6) + 17$ $(6,5)$

$36 - 48 + 17 = 5$

BEAT THE TEST!

1. A quadratic function $f(x)$ is shown.



Select symbols and values to restrict the domain of $f(x)$ so that $f^{-1}(x)$, is a function and the domain of $f(x)$ includes $x = -1$.

[− 2 , + ∞)

(	+	0	+	0	)
[	−	1	−	1	]
		2		2	
		3		3	
		4		4	
		5		5	
		∞		∞	