Section 5 – Topic 7
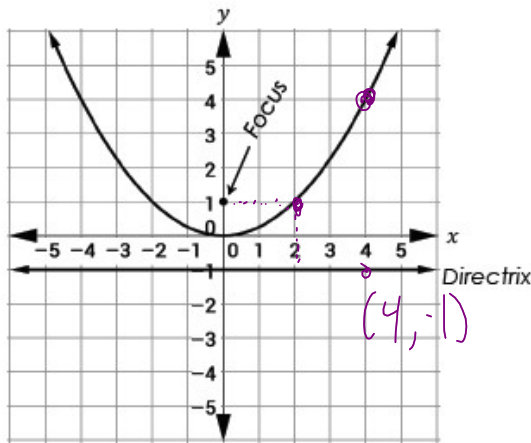## Writing Quadratic Equations when Given a Focus and Directrix

The following parabola represents a quadratic function.

Notice how the **focus** and **directrix** are labeled in the graph.

➤ A **parabola** is a set of points called a _locus_ of points.

➤ The _focus_ of a parabola is a fixed point on the interior of a parabola used in the formal definition of the curve.

➤ The _directrix_ of a parabola is a line perpendicular to the axis of symmetry used in the definition of a parabola.

Consider the point $(2, 1)$ on the parabola.

focus $(0, 1)$

What is the distance from this point to the focus?

2 units

What is the distance from this point to the directrix?

2 units

Now consider the point $(4, 4)$ on the same parabola.

What is the distance from this point to the focus? *Hint: Use the distance formula.* $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\sqrt{(4-0)^2 + (4-1)^2}$$
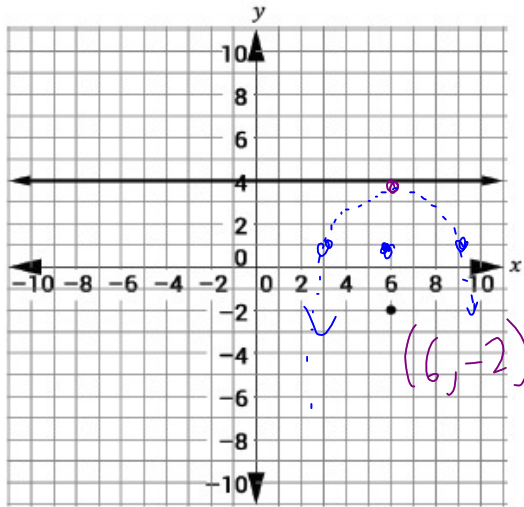$$\sqrt{(4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

What is the distance from this point to the directrix?

$$\sqrt{(1=4)^2 + (4+1)^2}$$

$$\sqrt{(5)^2} = 5$$

What do you notice about the distances?

They are equal

The graph below shows a focus and directrix for a specific parabola.

$(6, -2)$

What formula from geometry can help us find the vertex of the parabola?

Midpoint formula $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$

What is the vertex of the parabola?

$(6, 1)$

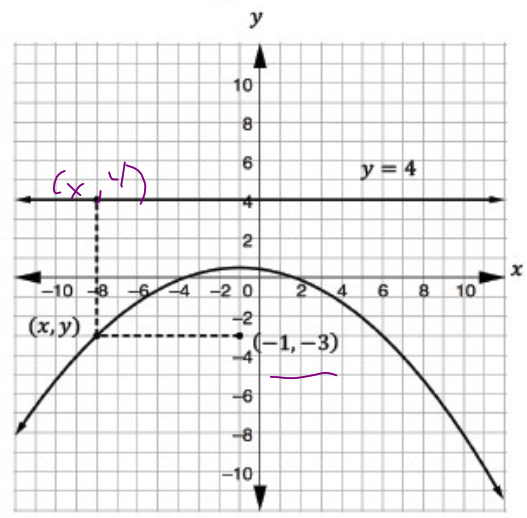$\left(\dfrac{6+6}{2}, \dfrac{4+(-2)}{2} \right)$

What other information would be needed to write the equation for the parabola?

$y = a(x-h)^2 + k$

$y = a(x-6)^2 + 1$

4

Let's Practice!

1. Consider the following parabola with the focus at $(-1,-3)$ and the directrix line at $y = 4$.

a. Use the distance formula to describe the distance from (x, y) to the focus.

$$\sqrt{(x+1)^2 + (y+3)^2}$$

b. Use the distance formula to describe the distance from (x, y) to the directrix.

$$\sqrt{(y-4)^2}$$

c. What do you know about these two distances?   they will be equal!

d. Use this information to write the equation for the quadratic.

$$\sqrt{(x+1)^2+(y+3)^2} = \sqrt{(y-4)^2}$$

$$(x+1)^2+(y+3)^2 = (y-4)^2$$

$$x^2+2x+1+y^2+6y+9 = y^2-8y+16$$

$$x^2+2x+6y+10 = -8y+16$$
$$+8y\ -10\quad +8y\ -10$$

$$x^2+2x+14y = 6$$

$$\frac{14y}{14} = \frac{-x^2-2x+6}{14}$$

$$y = -\frac{1}{14}x^2-\frac{1}{7}x+\frac{3}{7}$$

Try It!

2. A parabola has a focus on the origin and its directrix is $y = 7$.

$(0,0)$ $(0,7)$

What is the equation for the parabola?

$$\sqrt{(x-0)^2+(y-0)^2} = \sqrt{(y-7)^2}$$

$$x^2+y^2 = (y-7)^2$$

$$x^2+y^2 = y^2-14y+49$$

$$x^2 = -14y+49$$

$$\frac{14y = -x^2+49}{14}$$

$$y = -\frac{1}{14}x^2+\frac{7}{2}$$

BEAT THE TEST!

1. In the diagram below, F is the focus of the parabola, line d is the directrix, and $\overline{AB} \perp d$.
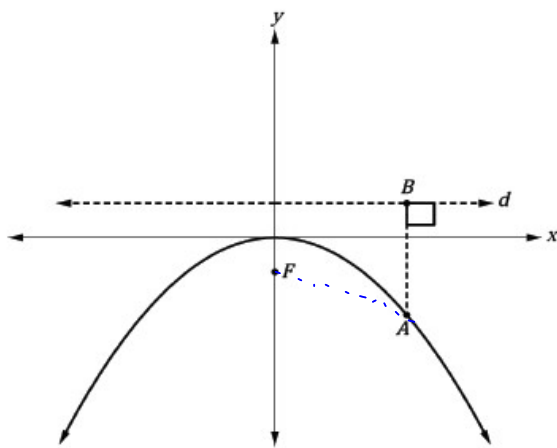
What is the relationship between FA and AB?

Ⓐ $FA < AB$
Ⓑ $FA = AB$
Ⓒ $FA > AB$
Ⓓ A relationship cannot be determined.

2. A parabola has a focus of $(2,5)$ and the directrix is $y = 3$.

    What is the equation for the parabola in standard form?