Section 5 – Topic 8
Systems of Equations with Quadratic Equations – Part 1

NASA launched a model rocket from the grounds of Cape Canaveral. The height, in feet, of the rocket with respect to time can be modeled by the function $h(x) = -x^2 + 10x$, where x is time, in seconds. At the same time, a Navy fleet shot a laser beam from a deck 14 feet above sea level. The laser beam follows a straight path represented by the equation $g(x) = -x + 14$.

$-x^2+10x=0$

$-x(x-10)=0$

$-x=0 \quad x-10=0$

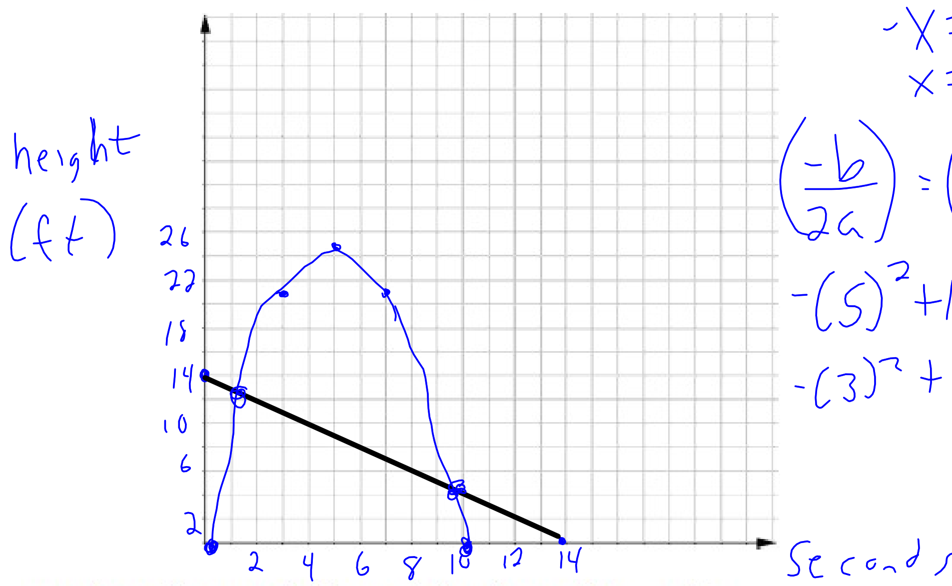
$x=0 \quad x=10$

$\left(\frac{-b}{2a}\right)=\left(\frac{-10}{2(-1)}\right)=5$

$-(5)^2+10(5)=-25+50=25$

$-(3)^2+10(3)=-9+30=21$

Model the situation on the graph below.



Mark on the graph the points where $f(x) = g(x)$.

What do these points represent? where the laser beam hits the rocket

The rocket and the laser beam model a quadratic-linear system of equations. We can also solve this system algebraically.

The following functions model the system.

$$f(x) = -x^2 + 10x$$
$$g(x) = -x + 14$$

One equation is a quadratic and has a degree of 2.

The other equation is a linear and has a degree of 1.

Where is $f(x) = g(x)$? Justify your answer algebraically.

$-x^2 + 10x = -x + 14$

$+x - 14 \quad +x - 14$

$-x^2 + 11x - 14 = 0$

$$\frac{-11 \pm \sqrt{11^2 - 4(-1)(-14)}}{2(-1)}$$

$$\frac{-11 \pm \sqrt{121 - 56}}{-2}$$

$$\frac{-11 \pm \sqrt{65}}{-2}$$

$$\frac{-11 \pm 8.1}{-2}$$

1.45

$$\frac{-11+8.1}{-2}$$

8.55

$$\frac{-11-8.1}{-2}$$

Does your algebraic answer support with your graphic solution? yes

Let's Practice!

1. Consider the following system.

$$f(x) = 3$$
$$g(x) = x^2 - 7x + 11$$

$0 = x^2 - 7x + 11$

Where is $f(x) = g(x)$? Justify your answer algebraically.

$x^2 - 7x + 11 = 3$ $\left(\frac{7}{2(1)}\right) = \frac{7}{2} = 3.5$

$x^2 - 7x + 8 = 0$

$\frac{7 \pm \sqrt{(-7)^2 - 4(1)(8)}}{2(1)}$

$\frac{7 \pm \sqrt{49 - 32}}{2}$

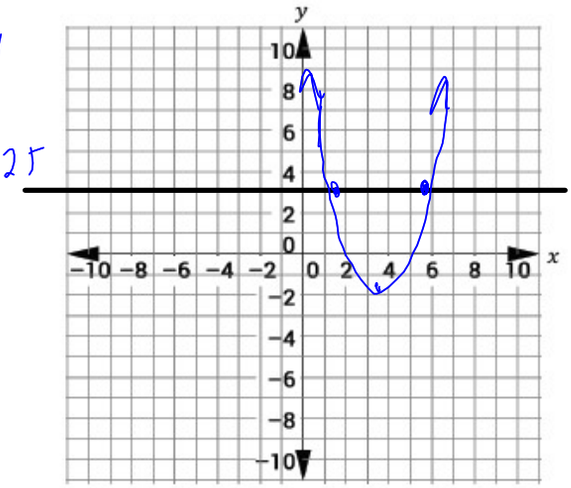
$\frac{7 \pm \sqrt{17}}{2}$ ≈ 5.6

$\frac{7 + 4.1}{2}$ $\frac{11.1}{2}$ $\frac{7 - 4.1}{2}$ $\frac{2.9}{2} = 1.5$ $(1.5, 3)$ $(5.6, 3)$

Graph the system. $(3.5, -1.25)$

$(3.5)^2 - 7(3.5) + 11$

$12.25 - 24.5 + 11 = -1.25$



2. Consider the following system.

$$\begin{cases} f(x) = x^2 - 2x - 5 \\ g(x) = -2x^2 + 5x + 1 \end{cases}$$

Where is $f(x) = g(x)$? Justify your answer algebraically.

$x^2 - 2x - 5 = -2x^2 + 5x + 1$

$+2x^2 - 5x - 1$

$3x^2 - 7x - 6 = 0$

$\frac{7 \pm \sqrt{(7)^2 - 4(3)(6)}}{2(3)}$

$\frac{7 \pm \sqrt{49 + 72}}{6}$

$\frac{7 \pm \sqrt{121}}{6}$

$\frac{7-11}{6} = \frac{-4}{6} = \frac{-2}{3}$

$\frac{7+11}{6} = \frac{18}{6} = 3$

$(3)^2 - 2(3) - 5$

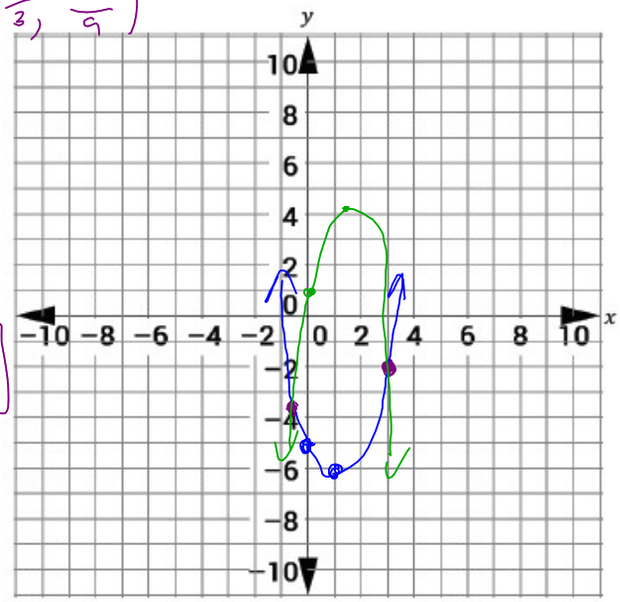
$9 - 6 - 5 = -2$ $(3, -2)$

$\left(-\frac{2}{3}\right)^2 - 2\left(-\frac{2}{3}\right) - 5$

$\frac{4}{9} + \frac{4}{3} - 5 \rightarrow \frac{4}{9} + \frac{12}{9} - \frac{45}{9}$

$\left(-\frac{2}{3}, -\frac{29}{9}\right)$

Graph the system.



$x^2 - 2x - 5$

$\frac{2}{2(1)} = 1$

$(1)^2 - 2(1) - 5$

$1 - 2 - 5 = -6$

$(1, -6)$

$-2x^2 + 5x + 1$

$\frac{-5}{2(-2)} = \frac{-5}{-4} = 1.25$

$-2(1.25)^2 + 5(1.25) + 1$

$-3.125 + 6.25 + 1 = 4.125$