# Section 6: Polynomial Functions
## Section 6 – Topic 1
## Classifying Polynomials and Closure Property

A polynomial is a finite sum of monomials.

Determine whether each of the following expressions is a polynomial. If the expression is not a polynomial, change the expression so that it is a polynomial.

polynomials can't have negative or fraction exponents

$3x^2 + 2y + 4$ yes

$8a^{\frac{1}{2}} + 2c$ no $8a^2 + 2c$

$5r + \frac{s}{t}$ $5r + st^{-1}$ no $\rightarrow$ $5r + st$

$\frac{5a + 4b}{2} = \frac{5a}{2} + \frac{4b}{2} = \frac{5a}{2} + 2b$ yes

$9x^{-3} + 2y + 7x^3$ no $9x^3 + 2y + 7x^3 = 16x^3 + 2y$

5 yes

We can classify polynomials by the number of terms.

| Number of Terms | Example | Name of Polynomial |
|---|---|---|
| 1 | $3x^7$ | monomial |
| 2 | $5m + 6n$ | binomial |
| 3 | $8r^2 + 4s + 7t$ | trinomial |
| 4 | $a + 2b + c^2 + 4$ | polynomial |

We can also classify polynomials by degree.

| Degree | Example | Type of Polynomial |
|---|---|---|
| 0 | $3$ | constant |
| 1 | $2x + 3z$ | linear |
| 2 | $3x^2 + 2y^2$ | quadratic |
| 3 | $6y^3$ | cubic |
| 4 | $5a^4 + 3b^3$ | quartic |

***Let's Practice!***

1. Describe two polynomial functions that we have seen so far.

$y = mx + b$ linear binomial

$y = ax^2 + bx + c$ quadratic trinomial

2. Explain if exponential functions are polynomial functions or not.

$y = a \cdot b^x$ - not a monomial (not a polynomial)

if x is in the exponential position, the function is not a polynomial

***Try It!***

Select the word that correctly completes each of the following statements.

3. A monomial is
   - ◉ always
   - ○ sometimes
   - ○ never

   a polynomial.

4. A polynomial is
   - ○ always
   - ◉ sometimes
   - ○ never

   a monomial.

5. A quadratic function is
   - ◉ always
   - ○ sometimes
   - ○ never

   a polynomial function.

We can also apply the ***Closure Property*** to polynomials.

A set is said to be closed under a specific mathematical operation if the result that occurs when you perform the operation on any two members of the set is also a member of the set.

Determine whether each of the following statements are true or false. If a statement is false, write a counterexample.

Integers are closed under addition.

True

Odd numbers are closed under addition.

False $11+11=22$

Even numbers are closed under addition.

True

Negative numbers are closed under multiplication.

False $(-5)(-5)=25$

Odd numbers are closed under multiplication. True

When referring to the Closure Property, what do you think "polynomials form a system analogous to the integers" means? Integers are closed in +, -, X, but not division. Same applies to polynomials

***Let's Practice!***

6. Determine whether each of the following statements is true or false. If the statement is false, write a counterexample.

   a. Polynomials are closed under addition.

   True

   b. Polynomials are closed under subtraction.

   True

***Try It!***

7. Determine whether each of the following statements is true or false. If the statement is false, write a counterexample.

   a. Polynomials are closed under multiplication.

   True

   b. Polynomials are closed under division.

   $\frac{2x}{2x^2} = \frac{1}{x}$ not a polynomial

## BEAT THE TEST!

1. Two functions are given below.

$$f(x) = x^3 + 2x^2 - 3x + 4$$
$$g(x) = x^2$$

Candice solved $\frac{f(x)}{g(x)}$ as follows:

$$\frac{x^3 + 2x^2 - 3x + 4}{x^2}$$

$$\frac{x^3}{x^2} + \frac{2x^2}{x^2} - \frac{3x}{x^2} + \frac{4}{x^2}$$

$$x + 2 - \frac{3}{x} + \frac{4}{x^2}$$

*Part A:* Candice's work illustrates that polynomials are

| | | |
|---|---|---|
| o closed<br>o not closed | under | o addition.<br>o division.<br>o multiplication.<br>o subtraction. |

*Part B:* Explain your answer from Part A.

She divided 2 polynomials, so the result was not a polynomial.