Section 2 – Topic 9
Systems of Linear Equations in Three Variables – Part 1

Suppose I have only quarters in my pocket. If I have 75 cents in change, can you tell me how many quarters are in my pocket?

Define a variable to represent the number of quarters.

q

Write an equation that models this situation.

$.25q = .75$

Solve the equation to find the number of quarters in my pocket.

$q = 3$

Suppose I have only quarters and dimes in my pocket. If I have **$1.50** in change, can you tell me how many quarters and dimes are in my pocket?

Suppose you have **9** coins in your pocket. How many are quarters? How many are dimes?

Define variables to represent the number of quarters and dimes.

q = quarters d = dimes

Write a system of equations that models this situation.

$(q + d = 9) = -.10$

$.25q + .10d = 1.50$

Solve the system to find the number of quarters and dimes in my pocket.

$-.10q - .10d = -.9$

$.25q + .10d = 1.50$

$.15q = .6$

$\frac{.15}{.15} = 4$ quarters

5 dimes

If we are going to solve a system with 3 unknowns, how many equations must we have?

three

Let's Practice!

1. Solve the following system of equations. (a, b, c)

$$a + b - 3c = -7$$
$$3b + c = -8$$
$$c = 1$$

$(-1, -3, 1)$

$3b + 1 = -8$
$-1 \quad -1$
$\frac{3b}{3} = \frac{-9}{3}$
$b = -3$

$a + (-3) - 3(1) = -7$
$a - 3 - 3 = -7$
$a - 6 = -7$
$+6 \quad +6$
$a = -1$

2. Solve the following system of equations.

(m, n, p)

$\left(\frac{1}{3}, 0, -2\right)$

A) $3m - 3n + p = -1$

B) $3m - n - p = 3$

C) $-6m + 4n + 3p = -8$

$3\left(\frac{1}{3}\right) - 3(0) + p = -1$

$1 - 0 + p = -1$

$1 + p = -1$

$-1 \quad -1$

$p = -2$

A + B

$3m - 3n + p = -1$

$3m - n - p = 3$

$6m - 4n = 2$

B + C

$(3m - n - p = 3)3$

$-6m + 4n + 3p = -8$

$9m - 3n - 3p = 9$

$3m + n = 1$

$6m - 4n = 2$

$(3m + n = 1)4$

$6m - 4n = 2$

$12m + 4n = 4$

$\frac{18m}{18} = \frac{6}{18}$

$m = \frac{1}{3}$

$3\left(\frac{1}{3}\right) + n = 1$

$1 + n = 1$

$n = 0$

Try It!

3. Solve the following system of equations:

$$\begin{aligned} 5a - 5b + 8c &= 6 \\ a + 2b + c &= 12 \\ 3b - c &= 10 \end{aligned}$$

(2, 4, 2)

$-5(a + 2b + c = 12)$

$a + 2(4) + 2 = 12$

$a + 8 + 2 = 12$

$a + 10 = 12$

$a = 2$

$5a - 5b + 8c = 6$

$-5a - 10b - 5c = -60$

$-15b + 3c = -54$

$(3b - c = 10)5$

$-15b + 3c = -54$

$15b - 5c = 50$

$-2c = -4$

$c = 2$

$3b - 2 = 10$

$3b = 12$

$b = 4$

Section 2 – Topic 10
Systems of Linear Equations in Three Variables – Part 2

Let's Practice!

1. An arena has **49,000** seats that sell for **\$25** on the ground level, **\$20** on the mid-level, and **\$15** on the upper-level. The number of seats on the upper-level equals the total number of seats on the ground level and mid-level combined. Suppose the arena brings in **\$882,500** from a sold-out event. How many seats does each level hold?

   Define variables to represent the number of seats in each level.

   g = ground u = upper
   m = middle

   Write a system of equations that models this situation.

   $g + m + u = 49{,}000$ $25g + 20m + 15u = 882{,}500$
   $u = g + m$

Use the system of equations to find the solution.

$g+m+(g+m)=49,000$

$25g+20m+15(g+m)=882,500$

$(2g+2m=49,000)\cdot -20$

$40g+35m=882,500$

$-40g-40m=-980,000$

$40g+35m=882,500$

$-5m=-97,500$

$\frac{-5m}{5}=\frac{-97,500}{5}$

$m=19,500$ seats

$2g+2(19,500)=49,000$

$2g+39,000=49,000$

$2g=10,000$

$g=5,000$ seats

$5,000+19,500+u=49,000$

$24,500+u=49,000$

$u=24,500$ seats

Try It!

2. SaraBeth is making custom jewelry to sell at the school carnival. She purchased round beads for **3** cents each, teardrop shaped beads for **5** cents each, and heart-shaped beads for **6** cents each. She bought twice as many round beads as heart shaped beads. SaraBeth bought a total of **450** beads and spent **$18.75**.

   a. Write a system of three equations that models how many beads of each type SaraBeth bought and the total amount she spent.

$r + t + h = 450$

$.03r + .05t + .06h = 18.75$

$r = 2h$

b. Solve your system using any method.

$2h + t + h = 450$

$.06h + .05t + .08h = 18.75$

$(3h + t = 450) \cdot .05$

$.12h + .05t = 18.75$

$2(125) = r$

250

round shape beads

$375 + t = 450$

$t = 75$

tear shaped

$-.15h - .05t = -22.5$

$12h + .05t = 18.75$

$-.03h = -3.75$

$-.03 \quad -.03$

$h = 125$ heartshape beads

c. How many of each type of bead did SaraBeth purchase?

BEAT THE TEST!

1. The school athletic director had a budget of **$500** to purchase **164** items for the soccer team. She purchased vests for **$2.50** each, soccer balls for **$9.25** each, and cones for **$0.75** each. She purchased **40** more cones than balls. How many of each item can she purchase?

Vests = 60

Soccer Balls = 32

Cones = 72

$v+b+c=164$

$b+40=c$

$2.50v+9.25b+.75(b+40)=500$

$v+2b+40=164$

$(v+2b=124)-5$

$-5v-10b=-620$

$2.50v+10b=470$

$-2.50v=-150$

$v=60$

$2.50v+9.25b+.75b+30=500$

$2.50v+10b=470$

$60+2b=124$

$2b=64$

$b=32$