Section 2 – Topic 6
Solving Linear Systems – Investigating Graphing, Substitution, and Elimination

Methods for solving systems of equations:

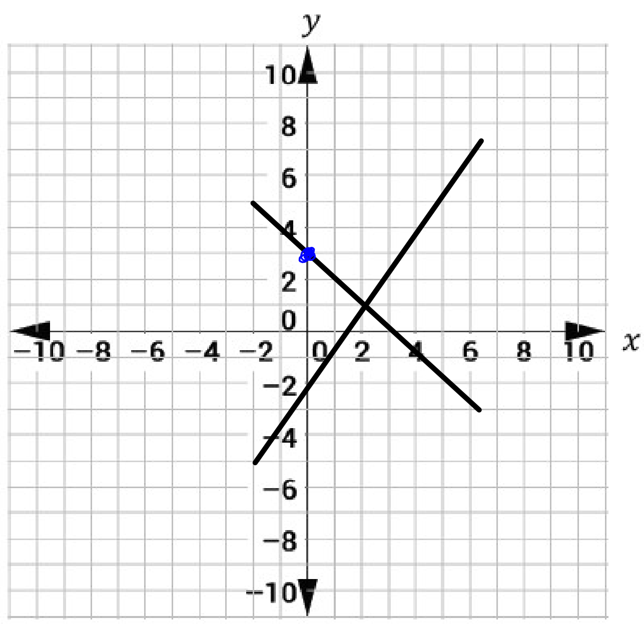
- Graphing
- Substitution
- Elimination
- Using a Table of Values
- Successive Approximations

Let's investigate solving systems by graphing to determine the nature of solutions to systems.

Sketch the graph of the following system:

$$x + y = 3$$
$$-3x + 2y = -4$$

$-3x + 2y = -4$
$+3x \quad +3x$
$\frac{2y}{2} = \frac{3x}{2} - \frac{4}{2}$
$y = \frac{3}{2}x - 2$



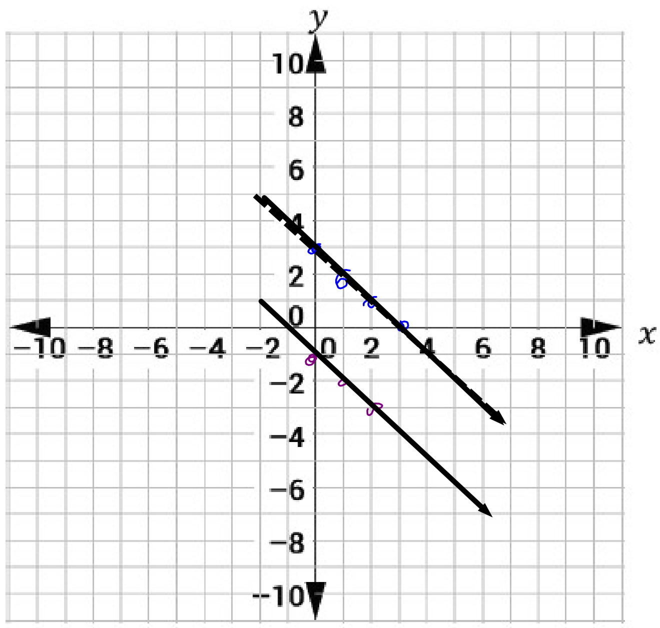
$x + y = 3$
$-x \quad -x$
$y = -x + 3$

What is/are the solution(s) to the system? $(2, 1)$

Suppose the second line of the original system is replaced with $2x + 2y = -2$.

Sketch the graph of the system.

$2x+2y=-2$
$-2x \quad -2x$
$\frac{2y}{2} = \frac{-2x}{2} \frac{-2}{2}$
$y=-x-1$



$y=-x+3$

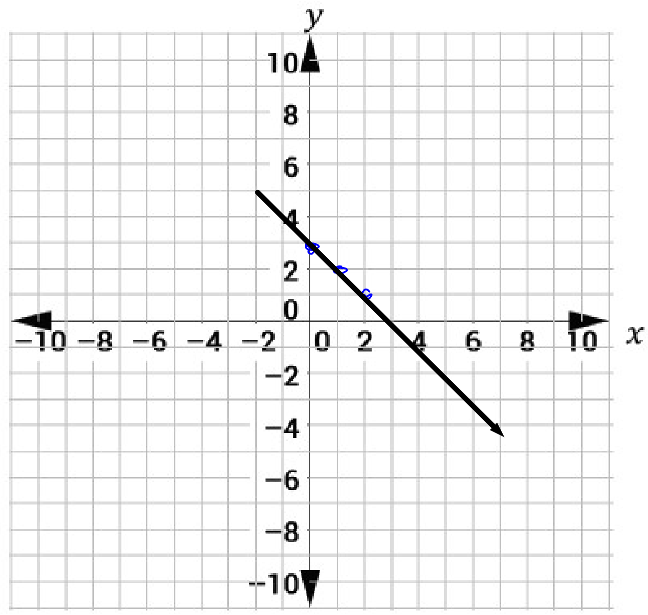
Make observations about the graph and the solution(s) to the new system. They are par

Suppose the second line of the original system is replaced with $-2x - 2y = -6$.

Sketch the graph of the system.

$y = -x + 3$

$-2x - 2y = -6$
$+2x \qquad +2x$
$\frac{-2y}{-2} = \frac{2x}{-2} - \frac{6}{-2}$
$y = -x + 3$



Make observations about the graph and the solution(s) to the new system. They are the same line
Infinitely Many Solutions (IMS)

Use your observations to make a conjecture about the solutions to systems of linear equations.

If the lines intersect, we have one solution (x, y)

If the lines are parallel, we have no solutions

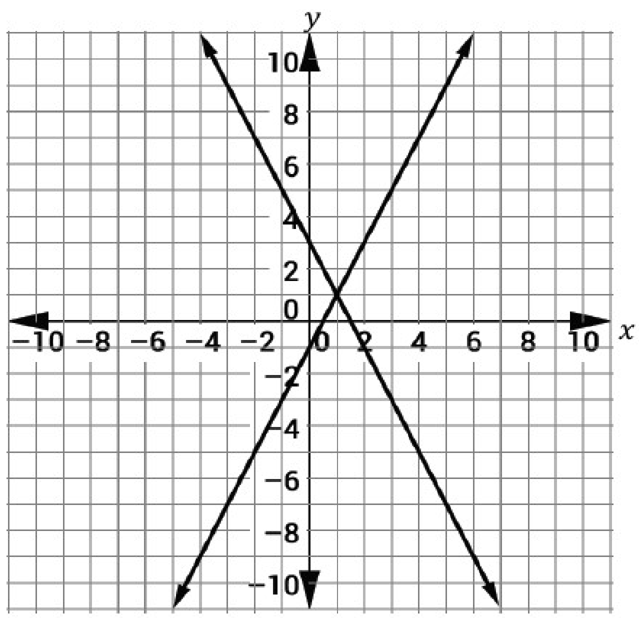
If the lines are identical, we have IMS

Let's investigate solving systems using substitution.

Consider the equations $y = f(x)$ and $y = g(x)$, where $f(x) = 2x - 1$ and $g(x) = -2x + 3$.

$y = 2x - 1$ $y = -2x + 3$

The graphical representation of the system is shown below.



Use the graph to find the solution to the system.

$(1, 1)$

Consider $f(x) = g(x)$. What is the solution for x?

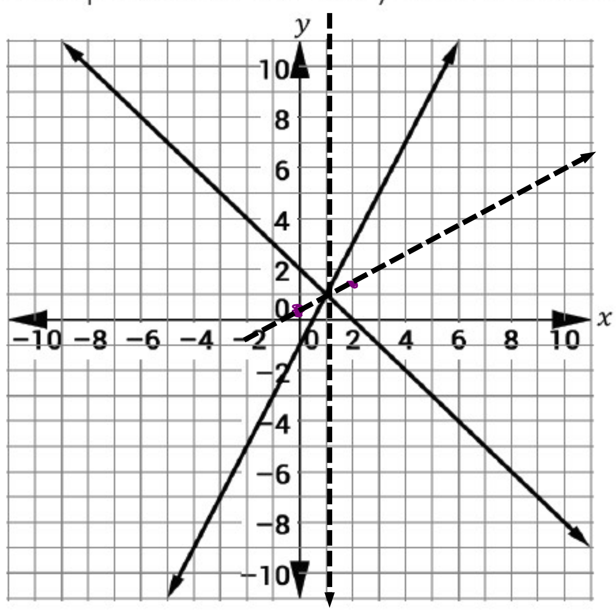
$$2x - 1 = -2x + 3$$
$$+2x \quad +2x$$
$$4x - 1 = 3$$
$$+1 \quad +1$$
$$4x = 4$$
$$x = 1$$

Let's investigate solving systems using elimination.

Consider the following system of equations.

$$y = 2x - 1$$
$$y = -x + 2$$

The graphical representation of the system is shown below.



(1, 1)

$x = 1$

$y = \frac{1}{2}x + \frac{1}{2}$

What is the resulting equation when you add the two equations?

What is the resulting equation when you add the two equations?

$$y = 2x - 1$$
$$(+)\ y = -x + 2$$
$$\frac{2y}{2} = \frac{x}{2} + \frac{1}{2}$$

$$y = \frac{1}{2}x + \frac{1}{2}$$

What is the resulting equation when you subtract the two equations?

$$\cancel{y} = 2x - 1$$
$$(-)\ \cancel{y} = -x + 2$$
$$0 = 3x - 3$$
$$+3 \qquad +3$$

$$3 = 3x$$
$$1 = x$$

Graph each of these equations on the previous graph.

Make observations from the graph and use those to explain why you can use the elimination method to solve.

The new equations intersect at solution already given

Let's Practice!

1. Consider the following system.

$$3x + 2y = 2$$
$$-3x + 2y = -4$$

Which method would you use to solve this system of equations? Explain your reasoning.

Elimination, because we can add the equation which will eliminate the x's.

Try It!

2. Consider the system of equations, $y = f(x)$ and $y = g(x)$, where $f(x) = 3x + 2$ and $g(x) = 5x + 4$.

   $y = 3x + 2$ $y = 5x + 4$

   Which method would you use to solve this system of equations? Explain your reasoning.

   Substitution, because we can set the equations equal to each other

BEAT THE TEST!

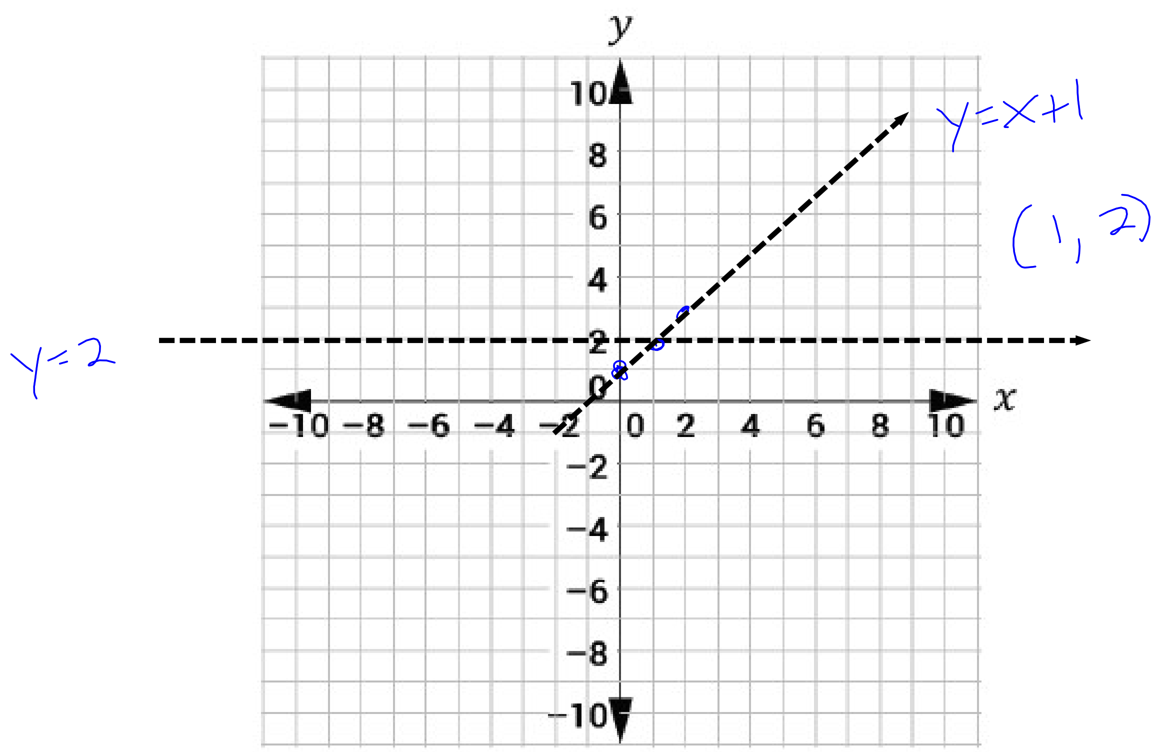
1. Consider the following system:

$$-2x + 5y = 8$$
$$2x + y = 4$$

$-\cancel{2x} + 5y = 8$
$(+)\ \cancel{2x} + y = 4$
$6y = 12$
$\frac{6y}{6} = \frac{12}{6}$
$y = 2$

$-2x + 5y = 8$
$(-)\ 2x + y = 4$
$-4x + 4y = 4$
$-x + y = 1$
$+x \quad +x$
$y = x + 1$

Part A: Find the solution by graphing the system.



$$-2x + 5y = 8 \Rightarrow \quad -2x+5y=8$$
$$2x + y = 4 \qquad +2x \quad +2x$$

$$\frac{5y}{5} = \frac{2x}{5} + \frac{8}{5} \qquad y = \frac{2}{5}x + \frac{8}{5}$$

Part B: Write an equation to replace the second equation so that the system will have infinitely many solutions. $\cdot 2$

Replace $2x+y=4$ with $-4x+10y=16$

Part C: Write an equation to replace the second equation so that the system will have no solution. $5y=2x+2$

Replace $2x+y=4$ with $-2x+5y=2$ $\quad y=\frac{2}{5}x+\frac{2}{5}$