Section 2 – Topic 6
Solving Linear Systems – Investigating Graphing, Substitution, and Elimination

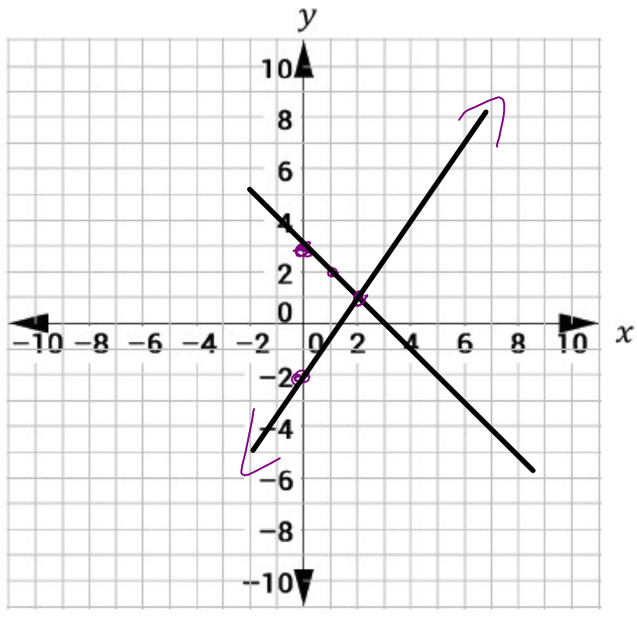
Methods for solving systems of equations:

- Graphing
- Substitution
- Elimination
- Using a Table of Values
- Successive Approximations

Let's investigate solving systems by graphing to determine the nature of solutions to systems.

Sketch the graph of the following system:

$$x + y = 3$$
$$-3x + 2y = -4$$



$x + y = 3$
$-x \quad -x$
$y = -x + 3$

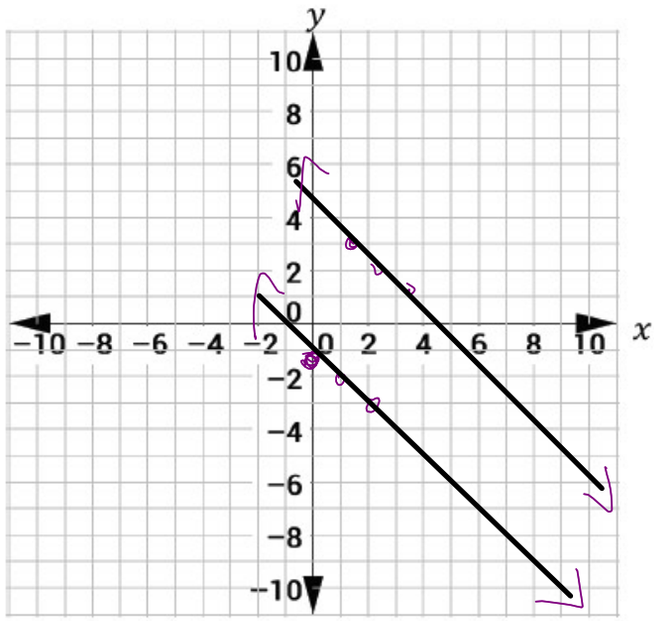
$-3x + 2y = -4$
$+3x \qquad +3x$
$\frac{2y}{2} = \frac{3x}{2} - \frac{4}{2}$
$y = \frac{3}{2}x - 2$

What is/are the solution(s) to the system? $(2, 1)$

Suppose the second line of the original system is replaced with $2x + 2y = -2$.

Sketch the graph of the system.

$y = -x + 3$



$2x + 2y = -2$

$-2x \quad -2x$

$\frac{2y = -2x - 2}{2}$

$y = -x - 1$

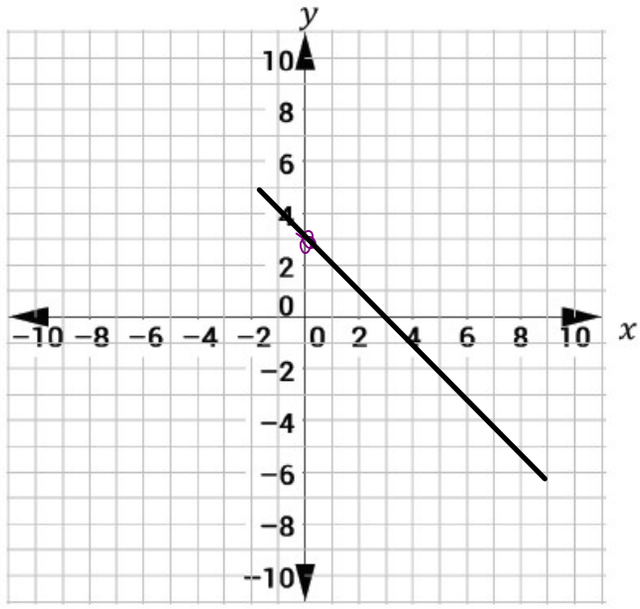
Make observations about the graph and the solution(s) to the new system.

parallel — no solution

Suppose the second line of the original system is replaced with $-2x - 2y = -6$.

Sketch the graph of the system.

$y = -x + 3$



IMS
Infinitely
Many
Solutions

$-2x - 2y = -6$
$+2x \quad +2x$

$\frac{-2y}{-2} = \frac{2x - 6}{-2}$

$y = -x + 3$

Make observations about the graph and the solution(s) to the new system. same line = same slope & y-int

Use your observations to make a conjecture about the solutions to systems of linear equations.

intersecting lines → one solution (x, y)

parallel lines → no solution

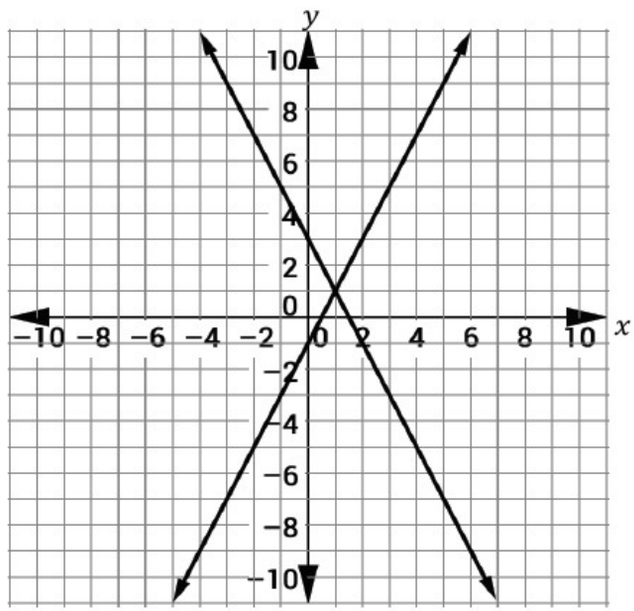
same line → IMS (Infinitely Many solutions)

Let's investigate solving systems using substitution.

Consider the equations $y = f(x)$ and $y = g(x)$, where $f(x) = 2x - 1$ and $g(x) = -2x + 3$.

$y = 2x - 1$ $y = -2x + 3$

The graphical representation of the system is shown below.



$2x - 1 = -2x + 3$

$+2x \quad +2x \quad +1$

$\frac{4x}{4} = \frac{4}{4}$ $x = 1$

$y = 2(1) - 1$

$y = 2 - 1$

$y = 1$ $(1, 1)$

Use the graph to find the solution to the system.

Consider $f(x) = g(x)$. What is the solution for x?

Let's investigate solving systems using elimination.

a) $y = \frac{1}{2}x + \frac{1}{2}$

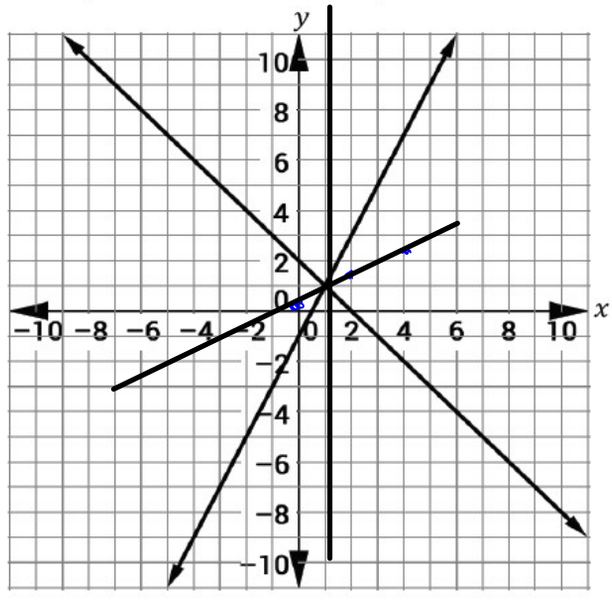
b) $x = 1$

$y = 2x - 1$
$y = -x + 2$ llowing system of equations.

$y = 2x - 1$
$2(y = -x + 2)$

$y = 2x - 1$
$+ \quad 2y = -2x + 4$
$3y = 3$

The graphical representation of the system is shown below.



$\frac{3y}{3} = \frac{3}{3}$ $y = 1$

(1, 1)

$1 = 2x - 1$
$+1 \quad +1$
$\frac{2}{2} = \frac{2x}{2}$ $x = 1$

What is the resulting equation when you add the two equations?

What is the resulting equation when you add the two equations?

$y = 2x - 1$
$y = -x + 2$

$\frac{2y}{2} = \frac{x}{2} + \frac{1}{2}$ $y = \frac{1}{2}x + \frac{1}{2}$

What is the resulting equation when you subtract the two equations?

$0 = 3x - 3$

$3 = 3x$ $x = 1$

Graph each of these equations on the previous graph.

Make observations from the graph and use those to explain why you can use the elimination method to solve.

Both equations intersect at the solution

Let's Practice!

1. Consider the following system.

$$3x + 2y = 2$$
$$-3x + 2y = -4$$

graphing
elimination
substitution

Which method would you use to solve this system of equations? Explain your reasoning.

Elimination, because the x's are inverse of each other and the terms are lined up in columns.

Try It!

2. Consider the system of equations, $y = f(x)$ and $y = g(x)$, where $f(x) = 3x + 2$ and $g(x) = 5x + 4$.

Which method would you use to solve this system of equations? Explain your reasoning.

Substitution, because both equations are set equal to the same variable.

BEAT THE TEST!

1. Consider the following system:

$$-2x + 5y = 8$$
$$2x + y = 4$$

$-2x + 5y = 8$

$+2x \quad +2x$

$\frac{5y}{5} = \frac{2x}{5} + \frac{8}{5}$

$y = \frac{2}{5}x + \frac{8}{5}$

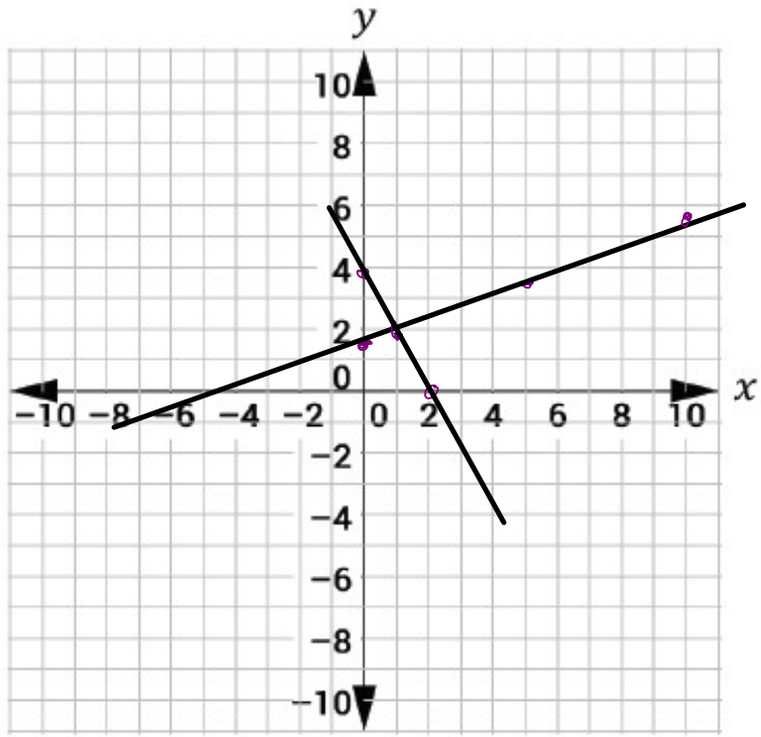
$2x + y = 4$

$-2x \quad -2x$

$y = -2x + 4$

Part A: Find the solution by graphing the system.

$y = \frac{2}{5}x + \frac{9}{5}$



$y = -2x + 4$

(1, 2)

$$-2x + 5y = 8$$
$$2x + y = 4$$

Part B: Write an equation to replace the second equation so that the system will have infinitely many solutions.

$-4x + 10y = 16$

Part C: Write an equation to replace the second equation so that the system will have no solution.

$-2x + 5y = 4$